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The offset-midpoint travel-time pyramid for P-wave in 2D transversely isotropic media with a tilted symmetry axis

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ABSTRACT

For pre-stack phase-shift migration in homogeneous isotropic media, the offset-midpoint travel time is represented by the double-square-root equation. The travel time as a function of offset and midpoint resembles the shape of Cheops' pyramid. This is also valid for transversely isotropic media with a vertical symmetry axis. In this study, we extend the offset-midpoint travel-time pyramid to the case of 2D transversely isotropic media with a tilted symmetry axis. The P-wave analytical travel-time pyramid is derived under the assumption of weak anelliptical property of the tilted transverse isotropy media. The travel-time equation for the dip-constrained transversely isotropic model is obtained from the depth-domain travel-time pyramid. The potential applications of the derived offset-midpoint travel-time equation include pre-stack Kirchhoff migration, anisotropic parameter estimation, and travel-time calculation in transversely isotropic media with a tilted symmetry axis.

Key words: Anisotropy, Modelling, Travel time, Migration.

INTRODUCTION

Transverse isotropy with a tilted symmetry axis (TTI) is a reasonable assumption to represent media with parallel tilted bedding in seismic exploration. Developing analytical travel-time formulations for pre-stack configuration for such media helps in many applications, including pre-stack Kirchhoff migration and anisotropic parameter estimation, etc.

For pre-stack migration in a homogeneous isotropic medium, the travel time as a function of offset and midpoint is given by a simple double-square-root (DSR) equation (Yilmaz 2001, pp. 628–639). Since the travel-time surface on the offset-midpoint plane from a scattering point is like a pyramid, Claerbout (1985, pp. 164–166) first named it as Cheops' pyramid. However, it is difficult to obtain such analytical travel-time formulations in anisotropic media since the explicit relation between group velocity and ray angle does not exist in anisotropic media. Even for transversely isotropic media with a vertical symmetry axis (VTI), the travel times are often calculated numerically. Alkhalifah (2000b) derived the offset-midpoint travel-time pyramid for VTI media through the perturbation method.

In this paper, we extend the offset-midpoint travel-time pyramid proposed by Alkhalifah (2000b) to the case of 2D TTI media. The TI symmetry axis is assumed to be located in the [x, z] plane. Under the assumption of the weak anelliptical property of TTI media, the horizontal slowness values for source and receiver are expanded in terms of anellipticity η. The accuracy of these expansions is enhanced using Shanks transformation (Bender and Orszag 1978, pp. 369–375). The travel-time pyramids are obtained both in depth and time domains. From the travel-time pyramid in the depth domain, we derive the reflection travel time for P-wave in a dip-constrained transversely isotropic (DTI) model. The detailed description of the DTI model is shown in Alkhalifah and Sava (2010, 2011).

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THE STATIONARY-PHASE METHOD

The 2D pre-stack phase-shift wavefield extrapolation defined in the offset-midpoint domain for a homogeneous isotropic medium (Yilmaz 2001, pp. 628–639) can be extended to the case of a homogeneous 2D general anisotropic medium,

$$P(x,h,z,t)=\int d\omega P(k_x,k_h,z=0,\omega)\int dk_h\int dk_x \exp(-i\omega\Upsilon), \quad (1)$$

where x, h, z, and t denote midpoint, source–receiver half-offset, depth, and time, respectively; k_x and k_h denote wavenumbers corresponding to x and h, respectively; $P(k_x,k_h,z=0,\omega)$ denotes the midpoint-offset seismic data in the frequency–wavenumber domain at the surface ($z=0$); $P(x,h,z,t)$ denotes the extrapolated seismic data in the time–space domain; and Υ denotes the travel-time shift given by

$$\Upsilon=-(q_s+q_g)z+2p_x x+2p_h h-t. \quad (2)$$

Here, $p_x=k_x/(2\omega)$ and $p_h=k_h/(2\omega)$ are the horizontal slowness values defined in the midpoint-offset space, and q_s and q_g are the vertical slowness values defined at the source and receiver positions, respectively.

To obtain the response of applying the pre-stack phase-shift wavefield extrapolation (1) on a single trace, we adopt the following definition (Alkhalifah 2000b),

$$P(x,h,z=0,t)=\tilde{P}(x,h,z=0,t)\delta(x-x_0,h-h_0), \quad (3)$$

where $\tilde{P}(x,h,z=0,t)$ denotes the time–space domain seismic record at the surface; $\delta(.,.)$ denotes Dirac delta function; and midpoint x_0 and half-offset h_0 correspond to the spatial position of the single trace.

By taking the Fourier transform (Yilmaz 2001, p. 156) of equation (3) and substituting it into equation (1), we obtain the single-trace response of the phase-shift migration operator given by

$$P(x,h=0,z,t=0) = \int d\omega \tilde{P}(x_0,h_0,z=0,\omega)\int dk_h\int dk_x \exp(i\omega T), \quad (4)$$

where T is the travel-time shift for single-trace migration,

$$T=(q_s+q_g)z-2p_x(x-x_0)+2p_h h_0. \quad (5)$$

According to the relations between horizontal slowness values for the source and receiver (Claerbout 1985, p. 181),

$$p_s=p_x-p_h, \quad (6)$$

$$p_g=p_x+p_h, \quad (7)$$

travel-time shift (5) is represented in terms of the slowness values for the source and receiver,

$$T=(q_s+q_g)z+p_s y_s+p_g y_g, \quad (8)$$

where q_s and q_g are vertical slowness values for the source and receiver, respectively, and $y_s=x_0-h_0-x$ and $y_g=x_0+h_0-x$ denote the lateral distances between the image point and the source and receiver, respectively.

Equation (8) represents the total travel-time shift of plane wave from the source located at $(x_0-h_0,0)$ to the image point located at (x,z) and back to the receiver located at $(x_0+h_0,0)$. Since the integrand given in equation (4) is an oscillatory function, the integral in k_x and k_h can be approximately calculated by a stationary-phase method. The biggest contribution from the integrand takes place when the oscillations are stationary and the phase function is either minimum or maximum (Alkhalifah 2000b). The stationary point corresponds to the extremum of equation (5). The travel time of seismic ray corresponds to the travel-time shift at the stationary point. From equations (4)–(8), it follows that the stationary point is determined from the following equation:

$$\frac{dT}{dp_{(s,g)}}=0, \quad (9)$$

which is the mathematical representation of Fermat's principle. By substituting travel-time shift (8) into equation (9), we obtain

$$\frac{dq_{(s,g)}}{dp_{(s,g)}}=-\frac{y_{(s,g)}}{z}, \quad (10)$$

where p, q, and y are either p_s, q_s, and y_s for the source or p_g, q_g, and y_g for the receiver, respectively. Equations (8) and (10) correspond to the exact travel time of seismic ray from the source to the image point then back to the receiver.

SLOWNESS APPROXIMATION AT STATIONARY POINT

In this section, we derive the analytical approximation of source and receiver slowness values at stationary point under the assumption of weak anelliptical property of TTI media. Equations (8) and (9) depend on both horizontal and vertical slowness projections. To obtain the slowness components for the source and receiver, we consider the slowness surface equation, which describes the relation between horizontal p and vertical q slowness projections. In practice, we can ignore the influence of SV-wave vertical velocity on P-wave velocity and travel time in VTI media (Tsvankin and Thomsen 1994; Alkhalifah 1998; Zhou and Greenhalgh

2008). The 2D slowness surface for P-wave in VTI media under acoustic approximation is given by Alkhalifah (1998, 2000a),

$$F_{VTI} = -2\eta\upsilon_0^2\upsilon_{nmo}^2 p_v^2 q_v^2 + \upsilon_0^2 q_v^2 + (1+2\eta)\upsilon_{nmo}^2 p_v^2 - 1 = 0, \quad (11)$$

where p_v and q_v are horizontal and vertical slowness values in VTI media, respectively. υ_0 and $\upsilon_{nmo} = \upsilon_0\sqrt{1+2\delta}$ are the vertical and normal moveout velocities, respectively. $\eta = (\varepsilon - \delta)/(1+2\delta)$ is the anellipticity parameter, and ε and δ are Thomsen (1986) anisotropy parameters.

For the 2D TTI media, the TI symmetry axis is located in the $[x, z]$ plane. The corresponding slowness surface is obtained by applying the following rotation operator,

$$p = p_v\cos\theta + q_v\sin\theta, \quad (12)$$

$$q = -p_v\sin\theta + q_v\cos\theta, \quad (13)$$

where p and q are the horizontal and vertical slowness values for P-wave in TTI media, respectively, and θ is the tilt of the TI symmetry axis measured from the vertical direction.

By substituting the expressions for p and q given in equations (12) and (13) into equation (10), we obtain the expression for dq_v/dp_v,

$$\frac{dq_v}{dp_v} = -\frac{y\cos\theta - z\sin\theta}{z\cos\theta + y\sin\theta} = -a, \quad (14)$$

where a new parameter a is defined.

From equations (11) and (14), we obtain the equations for p_v and q_v in a VTI medium, independently,

$$p_v^2\upsilon_{nmo}^4 - a^2\upsilon_0^2\left(-1+2p_v^2\upsilon_{nmo}^2\eta\right)^3\left(-1+p_v^2\upsilon_{nmo}^2(1+2\eta)\right) = 0, \quad (15)$$

$$a^2 q_v^2\upsilon_0^4 - \left(1 - q_v^2\upsilon_0^2\right)\upsilon_{nmo}^2\left(1+2\eta - 2q_v^2\upsilon_0^2\eta\right)^3 = 0. \quad (16)$$

Both equations (15) and (16) are quartic equations with respect to variables p_v^2 and q_v^2. Under the assumption of weak anelliptical property of TTI media, the value of anellipticity η is small. In this case, the solutions of equations (15) and (16) can be approximately represented in forms of second-order perturbation,

$$p_v = p_{v0} + p_{v1}(2\eta) + p_{v2}(2\eta)^2, \quad (17)$$

$$q_v = q_{v0} + q_{v1}(2\eta) + q_{v2}(2\eta)^2. \quad (18)$$

By substituting equation (17) into equation (15), we obtain a polynomial equation in η. Since equation (17) is the trial solution of equation (15), it follows that all coefficients in this polynomial equation should be zero. Consequently, we derive

$$p_{v0} = \frac{a\upsilon_0}{\upsilon_{nmo}\sqrt{a^2\upsilon_0^2 + \upsilon_{nmo}^2}}, \quad (19)$$

$$p_{v1} = -\frac{a^3\upsilon_0^3\left(a^2\upsilon_0^2 + 4\upsilon_{nmo}^2\right)}{2\upsilon_{nmo}\left(a^2\upsilon_0^2 + \upsilon_{nmo}^2\right)^{5/2}}, \quad (20)$$

$$p_{v2} = \frac{3\upsilon_0^5(a^9\upsilon_0^4 + 4a^7\upsilon_0^2\upsilon_{nmo}^2 + 24a^5\upsilon_{nmo}^4)}{8\upsilon_{nmo}(a^2\upsilon_0^2 + \upsilon_{nmo}^2)^{9/2}}. \quad (21)$$

In a similar way, we obtain the corresponding coefficients q_{vi}, $i = 0, 1, 2$,

$$q_{v0} = \frac{\upsilon_{nmo}}{\upsilon_0\sqrt{a^2\upsilon_0^2 + \upsilon_{nmo}^2}}, \quad (22)$$

$$q_{v1} = \frac{3a^4\upsilon_0^3\upsilon_{nmo}}{2\left(a^2\upsilon_0^2 + \upsilon_{nmo}^2\right)^{5/2}}, \quad (23)$$

$$q_{v2} = \frac{3\upsilon_0^5\upsilon_{nmo}\left(-a^8\upsilon_0^2 + 20a^6\upsilon_{nmo}^2\right)}{8(a^2\upsilon_0^2 + \upsilon_{nmo}^2)^{9/2}}. \quad (24)$$

Substitution of equations (17) and (18) into equation (12) results in the analytical approximation of horizontal slowness in a TTI medium,

$$p = p_0 + p_1(2\eta) + p_2(2\eta)^2, \quad (25)$$

where coefficients p_i, $i = 0, 1, 2$, are the linear combination of p_{vi} and q_{vi}, given by

$$p_i = p_{vi}cos\theta + q_{vi}sin\theta, \; i = 0, 1, 2. \quad (26)$$

Here, the expressions for p_{vi} and q_{vi}, $i = 0, 1, 2$, are given in equations (19)–(24). Shanks transformation (Bender and Orszag 1978, pp. 369–375) is employed to improve the accuracy of approximation (25). The final approximation for p takes the form

$$p = p_0 + \frac{2p_1^2\eta}{p_1 - 2p_2\eta}. \quad (27)$$

The horizontal slowness values at source and receiver locations can be calculated using this equation. Note that squaring equation (25) and setting the tilt to zero followed by Shanks transformation results in the horizontal slowness approximation for VTI media in Alkhalifah (2000b). The detailed derivation is shown in Appendix A.

The vertical slowness for TTI media can also be obtained in the similar way. However, a much more accurate

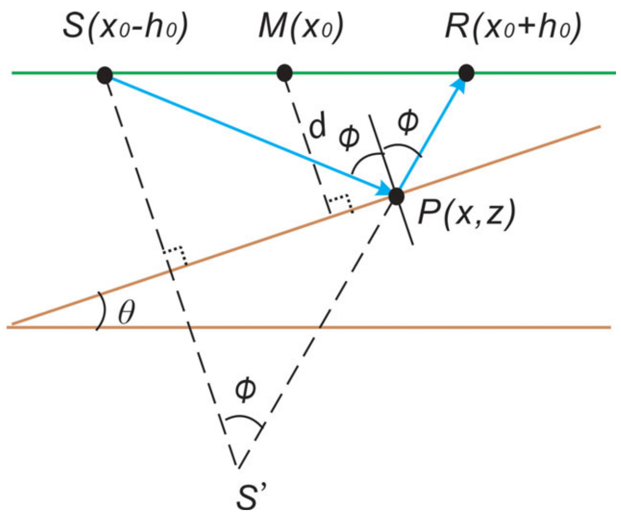

Figure 1 Schematic plot of the midpoint-offset domain in a DTI model. The TI symmetry axis is normal to the reflector. x_0 denotes the lateral position of source. h_0 denotes half-offset. θ denotes the dip of the reflector. $d = \tau \upsilon_0/2$ denotes the normal depth of the reflector under common midpoint M, where τ denotes the two-way zero-offset travel time at midpoint M, and υ_0 denotes the symmetry-direction P-wave velocity in the DTI model. The seismic ray propagates from source S to point P and reflects back to receiver R. Φ denotes the angle between incident (or reflected) ray and normal of the reflector. For a specified offset, the angles of reflection and incidence are equal.

approximation of the vertical slowness in TTI media was developed by Stovas and Alkhalifah (2012),

$$q = q_0(p) + \frac{2q_1^2(p)\eta}{q_1(p) - 2q_2(p)\eta}, \tag{28}$$

where coefficients q_i, $i = 0, 1, 2$, are the first- and second-order perturbation coefficients (see Appendix B).

Equation (28) is used to calculate vertical slowness values for the source and the receiver. The functional forms for $q\,(p)$ taken at source and receiver positions are different since the slowness surface for a TTI medium is non-symmetric with respect to the vertical axis (Golikov and Stovas 2012a, b; Stovas and Alkhalifah 2012). For a given horizontal slowness value p, we can compute two vertical slowness values q corresponding to down-going and up-going waves from equation (28). Down-going wave should be selected for p_s, and up-going wave should be for p_g.

DEPTH- AND TIME-DOMAIN TRAVEL-TIME PYRAMIDS

Substituting expressions for q_s and q_g given in equation (28) into equation (8), we obtain the depth-domain offset-midpoint travel-time pyramid for TTI media,

$$T(x, x_0, h_0, z) = \left(q_{s0} + \frac{2q_{s1}^2\eta}{q_{s1} - 2q_{s2}\eta} + q_{g0} + \frac{2q_{g1}^2\eta}{q_{g1} - 2q_{g2}\eta} \right) z + p_s y_s + p_g y_g, \tag{29}$$

where the horizontal slowness values p_s and p_g can be evaluated through equation (27), and the coefficients for vertical slowness q_{si} and q_{gi}, $i = 0, 1, 2$, are derived by Stovas and Alkhalifah (2012), as shown in Appendix B. The expressions for y_s and y_g are shown after equation (8). Equation (29) is also called as Cheops' pyramid by referring to Claerbout (1985, pp. 164–166) and Alkhalifah (2000b).

To obtain the time-domain travel-time pyramid, we consider the case that half-offset is equal to zero ($h_0 = 0$) and lateral coordinate of image point is equal to lateral midpoint coordinate ($x = x_0$). From equation (29), it follows that the zero-offset two-way travel time τ is given by

$$\tau = T(x, x_0 = x, h_0 = 0, z) = 2q_{z0}z, \tag{30}$$

where

$$q_{z0} = \frac{1}{2}\left(q_{s0} + \frac{2q_{s1}^2\eta}{q_{s1} - 2q_{s2}\eta} + q_{g0} + \frac{2q_{g1}^2\eta}{q_{g1} - 2q_{g2}\eta} \right)\Bigg|_{x=x_0, h_0=0} \tag{31}$$

denotes the vertical slowness for the zero-offset seismic ray. For a VTI medium, q_{z0} is reduced to $1/v_0$.

By substituting equation (30) into equation (29), we derive the time-domain offset-midpoint travel-time pyramid for P-wave in a TTI medium,

$$T(x, x_0, h_0, \tau) = \frac{1}{2q_{z0}}\left(q_{s0} + \frac{2q_{s1}^2\eta}{q_{s1} - 2q_{s2}\eta} + q_{g0} + \frac{2q_{g1}^2\eta}{q_{g1} - 2q_{g2}\eta} \right)\tau + p_s y_s + p_g y_g. \tag{32}$$

TRAVEL-TIME CALCULATION FOR REFLECTED P-WAVE IN A DIP-CONSTRAINED TRANSVERSELY ISOTROPIC MODEL

The depth-domain offset-midpoint travel-time pyramid equation derived above can be used to obtain the travel-time equation for a 2D DTI model. A DTI model is a type of transversely isotropic medium with symmetry axis perpendicular to a dipping reflector (Alkhalifah and Sava 2010, 2011). Figure 1 shows a 2D DTI model with dip θ. From

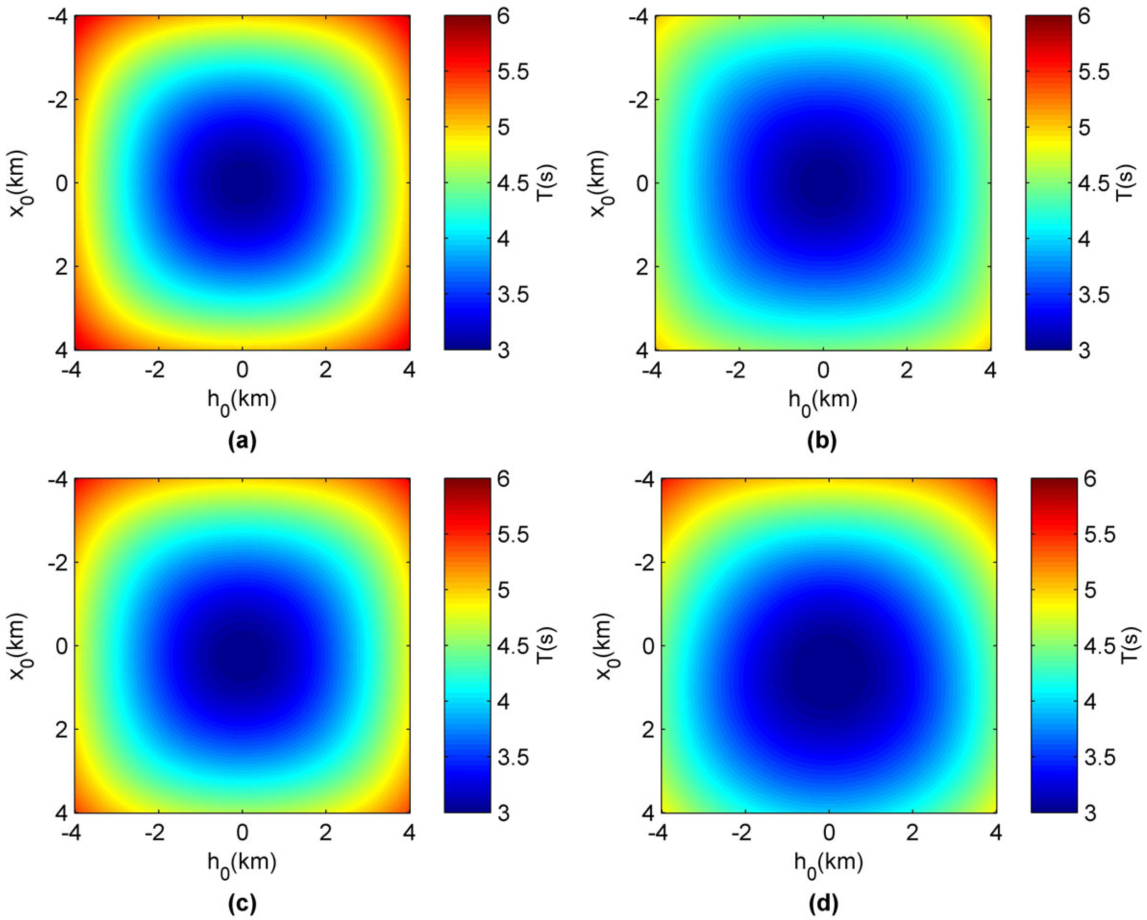

Figure 2 Travel time as a function of half offset h and midpoint x_0 for an isotropic medium (a), a VTI medium ($\eta = 0.2$, $\delta = 0.1$) (b), an elliptical TTI medium ($\eta = 0$,$\delta = 0.1$,$\theta = \pi/6$) (c), and a TTI medium ($\eta = 0.2$,$\delta = 0.1$,$\theta = \pi/6$) (d). υ_0 is $2km/s$. The zero-offset two-way travel time τ is $3s$. All plots are shown under the same color scale.

the geometrical relations shown in Fig. 1, we derive the coordinates of reflection point P,

$$x - x_0 = \frac{\left(\tau^2\upsilon_0^2 + 4h_0^2\cos^2\theta\right)\sin\theta}{2\tau\upsilon_0}, \tag{33}$$

$$z = \frac{\left(\tau^2\upsilon_0^2 - 4h_0^2\sin^2\theta\right)\cos\theta}{2\tau\upsilon_0}. \tag{34}$$

From the depth-domain travel-time pyramid (29), it follows that the P-wave reflection travel time in a DTI model is

$$T = \frac{\tau\upsilon_0}{2}\left(1 - \frac{4h_0^2}{\tau^2\upsilon_0^2}\sin^2\theta\right)\cos\theta\left(\frac{q_{s0}q_{s1} + (q_{s1}^2 - q_{s0}q_{s2})\eta}{q_{s1} - q_{s2}\eta} + \frac{q_{g0}q_{g1} + (q_{g1}^2 - q_{g0}q_{g2})\eta}{q_{g1} - q_{g2}\eta}\right) + p_s y_s + p_g y_g, \tag{35}$$

where the expressions for p_s and p_g are given in equation (27), and y_s and y_g can be obtained by inserting equation (33) to their expressions below equation (8). For the case of a DTI model with the elliptical property ($\eta = 0$), equation (35) is reduced to

$$T = \tau\sqrt{1 + \frac{4h^2\cos^2\theta}{\tau^2\upsilon_{nmo}^2}} \tag{36}$$

Further simplification to the isotropic case results in the same equation but with $\upsilon_{nmo} = \upsilon_0$.

NUMERICAL EXAMPLES

To check the shape of the travel-time pyramid as a function of offset and midpoint, we compare the travel-time pyramid in a TTI model with the ones from isotropic, VTI, and tilted elliptical isotropic (TEI) models. All four models have the same zero-offset two-way travel time, i.e., $\tau = 3s$. The P-wave velocity in the isotropic model is $2.0km/s$, which is also adopted as the

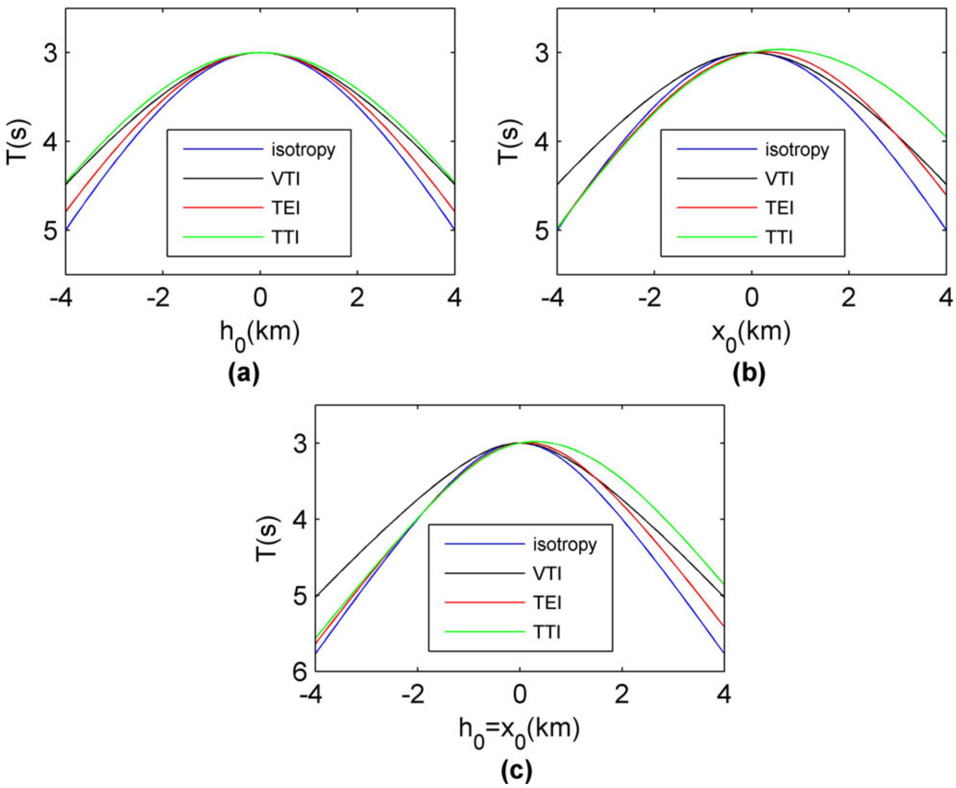

Figure 3 Comparison of travel times extracted from travel-time pyramids for $x_0 = 0$ (a), $h_0 = 0$ (b), and $x_0 = h_0$ (c) in Fig. 2.

P-wave velocity along the symmetry axis for all anisotropic models.

Figure 2 shows travel-time pyramids calculated from equation (32) for isotropic, VTI, TEI, and TTI models. The lateral location of the image point is taken as $x = 0$. Figure 3 shows the travel-time curves extracted from the travel-time pyramids in Fig. 2 for $x_0 = 0$, $h_0 = 0$, and $x_0 = h_0$. From Figs. 2 and 3, we can see that the travel-time pyramids for isotropic and VTI models are symmetric with respect to both midpoint and half offset. By contrast, the travel-time pyramids for TEI and TTI models are non-symmetric with respect to midpoint position. For both models, the peaks of travel-time pyramids are shifted towards the dipping direction of the TI symmetry axis. Compared with the travel-time pyramid in the TEI model, this behavior is much more obvious for the travel-time pyramid in the TTI model. This is because the group velocity surface is not symmetric with respect to the vertical axis, and the anisotropy parameters control the shape of the travel-time surfaces.

Next, we employ the time-domain travel-time pyramid (29) to investigate the variation of the common-offset migration isochrone versus the tilt of the TI symmetry axis. The TTI model in the first example is adopted again. To obtain the common-offset migration isochrone, we assume that both half-offset $h_0 = 0.5km$ and two-way travel time $t = 3s$ are unchanged, representing a position in the data space. The spatial positions of all image points are determined by solving equation (32). Figure 4 shows the influence of the tilt of the TI symmetry axis on the migration isochrones. We can see that the migration isochrone becomes much more non-symmetric with respect to the vertical direction with increase in the tilt of the TI symmetry axis. It means that the ignorance of tilt may lead to significant error in TTI pre-stack migration.

In the third example, we apply travel-time equation (35) to study the influence of dip angle in DTI model on the reflection P-wave travel time and test its accuracy. The zero-offset two-way travel time is $\tau = 3s$. From Fig. 5, we can see that the travel time for a constant non-zero offset becomes small with the increase in dip for the DTI model. Figure 6 shows the relative errors of travel-time approximation (35). The exact travel time is calculated by two-point ray tracing method (see Appendix C). We can see that equation (35) is accurate enough for practical application even for large values of dip angle since the maximum relative error is no more than 0.08%.

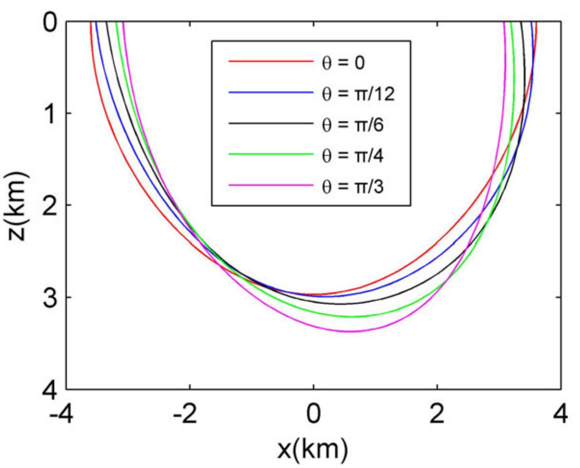

Figure 4 Migration isochrones in TTI media. The position of the midpoint is fixed. The source–receiver offset and travel time are 1*km* and 3*s*, respectively. The midpoint is located at x=0.

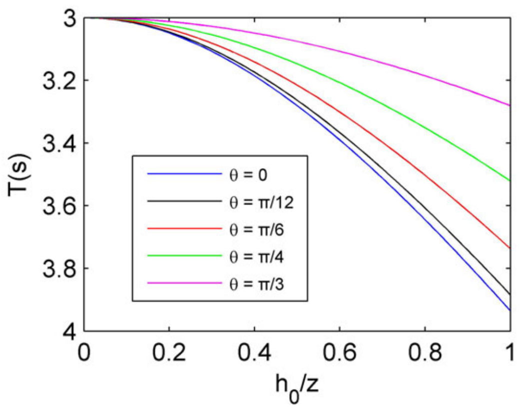

Figure 5 Travel-time curves as a function of the ratio of half-offset to depth for various reflector dip angles in a DTI model ($v_0 = 2km/s$, $\delta = 0.1$, $\eta = 0.2$).

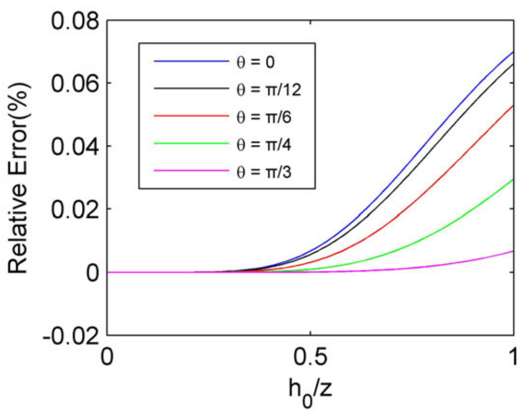

Figure 6 The relative error of expression (35) as a function of half-offset-to-depth ratio for different dip angles. The parameters of DTI model are the same as for Fig. 5.

DISCUSSIONS

The offset-midpoint travel-time pyramid (29) denotes the P-wave diffraction travel time from a single scattering point in a 2D TTI medium. Diffraction travel time is used for diffraction-based seismic imaging (e.g., Moser and Howard 2008; Waheed *et al*. 2013) and local velocity analysis (e.g., Dell *et al*. 2013). Investigating the sensitivity of travel time to model parameter is an important step for travel-time-based seismic inversion (Chapman and Miller 1996; Zhou and Greenhalgh 2008). Our first numerical example illustrates that the travel-time pyramid can at least help realize this point for seismic inversion based on diffraction travel time in TTI media.

The multiple-trace phase-shift migration corresponds to the sum of equation (4) over midpoint x_0 and offset h_0. With the aid of stationary-phase method, we can realize the offset-midpoint domain pre-stack migration in a homogeneous 2D TTI medium by equations (4) and (29). For vertically inhomogeneous 2D TTI media, the vertically inhomogeneous medium can be assumed to be locally 'homogeneous' for each extrapolation step Δz. In each extrapolation step Δz, seismic migration is realized by equations (4) and (29).

CONCLUSIONS

The analytical offset-midpoint travel-time pyramid for P-wave TTI media has been derived under the assumption of weak anelliptical property of TTI media. The perturbation method and Shanks transformation are utilized to obtain a relatively simple analytical form for the horizontal and vertical slowness values for the source and receiver. The depth-domain offset-midpoint travel-time pyramid is employed to derive the travel time of reflected P-wave in a DTI model. The potential applications of the derived offset-midpoint travel-time equations are the pre-stack Kirchhoff migration in 2D TTI media, sensitivity analysis of P-wave diffraction travel time to TTI model parameters, and the forward modeling of P-wave reflection travel time for the DTI model parameter inversion.

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APPENDIX A

Derivation of Alkhalifah's approximation for squared horizontal slowness in a transversely isotropic medium with a vertical symmetry axis

In this appendix, we derive Alkhalifah's (2000b) horizontal slowness approximation from slowness perturbation (25) for VTI media. Squaring equation (17) and keeping terms up to the second order with respect to the anelliptic parameter η leads to the approximation of horizontal slowness squared,

$$p_v^2 = p_{v0}^2 + 2p_{v0}p_{v1}(2\eta) + (p_{v1}^2 + 2p_{v0}p_{v2})(2\eta)^2. \quad \text{(A1)}$$

Shanks transformation is applied to improve the accuracy of equation (A-1). The final form is

$$p_v^2 = p_{v0}^2 + \frac{4p_{v0}^2 p_{v1}^2 \eta}{p_{v0}p_{v1} - \left(p_{v1}^2 + 2p_{v0}p_{v2}\right)\eta}. \quad \text{(A2)}$$

In case that tilt angle θ is equal to zero, equation (14) becomes $a = y/z$. From the relation between depth z and zero-offset two-way travel time τ, we derive the approximation for the horizontal slowness squared in a VTI medium (Alkhalifah 2000b),

$$p_v^2 = \frac{y^2\left(y^6 + 6\upsilon_{nmo}^2 y^4(1-\eta)\tau^2 + 3\upsilon_{nmo}^4 y^2(3+4\eta)\tau^4 + 4\upsilon_{nmo}^6\tau^6\right)}{\upsilon_{nmo}^2\left(y^2+\upsilon_{nmo}^2\tau^2\right)\left(y^6(1+2\eta) + 2\upsilon_{nmo}^2 y^4(3+5\eta)\tau^2 + \upsilon^4 y^2(9+44\eta)\tau^4 + 4\upsilon_{nmo}^6\tau^6\right)}. \quad \text{(A3)}$$

APPENDIX B

Slowness surface approximation for transversely isotropic media with a tilted vertical symmetry axis

Stovas and Alkhalifah (2012) derived the vertical slowness approximation (28) for TTI media by combining the perturbation theory with respect to the anelliptic parameter η and Shanks transformation. The coefficient q_j, $j = 0, 1, 2$, in equation (28) is the function of horizontal slowness p,

$$q_0^{(\pm)}(p) = \frac{p\left(\upsilon_{nmo}^2 - \upsilon_0^2\right)\sin\theta\cos\theta \pm \sqrt{V^2(\theta) - p^2\upsilon_0^2\upsilon_{nmo}^2}}{V^2(\theta)}, \quad \text{(B1)}$$

$$q_1^{(\pm)}(p) = \mp\upsilon_{nmo}^2 \frac{\left(P^{(\pm)}(p,\theta)\right)^2\left(1 - \upsilon_0^2\left(Q^{(\pm)}(p,\theta)\right)^2\right)}{2\sqrt{V^2(\theta) - p^2\upsilon_0^2\upsilon_{nmo}^2}}, \quad \text{(B2)}$$

$$q_2^{(\pm)}(p) = \mp q_1^{(\pm)} \frac{q_1^{(\pm)}V^2(p,\theta) - 2\upsilon_{nmo}^2 P^{(\pm)}(p,\theta)F^{(\pm)}(p,\theta)}{2\sqrt{V^2(p,\theta) - p^2\upsilon_0^2\upsilon_{nmo}^2}}, \quad \text{(B3)}$$

with

$$V^2(\theta) = \upsilon_0^2\cos^2\theta + \upsilon_{nmo}^2\sin^2\theta, \quad \text{(B4)}$$

$$P^{(\pm)}(p,\theta) = p\cos\theta - q_0^{(\pm)}\sin^2\theta, \quad \text{(B5)}$$

$$Q^{(\pm)}(p,\theta) = p\sin\theta + q_0^{(\pm)}\cos\theta, \quad \text{(B6)}$$

$$F^{(\pm)}(p,\theta) = \sin\theta + \upsilon_0^2 Q^{(\pm)}(p,\theta)P^{(\pm)}(p,2\theta), \quad \text{(B7)}$$

where $q_j^{(\pm)}(p)$, $j = 0, 1, 2$ are the antisymmetric functions for the upper (+) and lower (−) parts of the slowness surface; $V(\theta)$ is the elliptic velocity (Golikov and Stovas 2012a); θ is the tilt angle of the symmetry axis; $P^{(\pm)}(p,\theta)$ is the rotated horizontal slowness; $Q^{(\pm)}(p,\theta)$ is the rotated vertical slowness; and $F^{(\pm)}(p,\theta)$ is the antisymmetric term. In particular, $q_0^{(\pm)}(p)$ defines the elliptical TTI slowness surface (Golikov and Stovas 2012a). Note the following symmetry relations: $q_j^{(+)}(-p) = -q_j^{(-)}(p)$, $j = 0, 1, 2$.

APPENDIX C

Two-point ray tracing for P-wave in a 2D homogeneous dip-constrained transversely isotropic model

For the DTI (Alkhalifah and Sava 2010) model shown in Fig. 1, the exact midpoint-offset travel time is written as

$$T = \frac{2h_0\cos\theta}{V_g(\phi)\sin\phi}, \quad \text{(C1)}$$

where h_0 denotes source–receiver half-offset; $V_g(\phi)$ denotes the group velocity for reflection angle ϕ; and θ denotes the dip of this DTI layer.

From geometrical relations in Fig. 1, we derive the expression for $\sin\phi$ given by

$$\sin\phi = \frac{2h_0\cos\theta}{\sqrt{(2h_0\cos\theta)^2 + (\tau\upsilon_0)^2}}, \quad \text{(C2)}$$

where τ is the two-way zero-offset travel time, υ_0 denotes the P-wave velocity along the symmetry axis of TTI media.

The projections of group velocity (V_{gx}, V_{gz}) in a 2D VTI medium (Tsvankin 2001, p. 6) read

$$V_{gx} = \upsilon \sin\theta + \frac{d\upsilon}{d\theta}\cos\theta, \tag{C3}$$

$$V_{gz} = \upsilon \cos\theta - \frac{d\upsilon}{d\theta}\sin\theta, \tag{C4}$$

where θ denotes the phase angle of slowness measured from the TI symmetry axis, and υ denotes the phase velocity. For P-wave in an acoustic VTI medium, the expression for υ is obtained from Alkhalifah (1998),

$$2\upsilon^2(\theta) = \upsilon_0^2\cos^2\theta + (1+2\eta)\upsilon_n^2\sin^2\theta + \sqrt{(\upsilon_0^2 cos^2\theta + (1+2\eta)\upsilon_n^2 sin^2\theta)^2 - 2\eta\upsilon_0^2\upsilon_n^2 sin^2 2\theta}, \tag{C5}$$

where υ_0 denotes the velocity for P-wave along the VTI symmetry axis; υ_n denotes the normal moveout velocity for VTI media; $\upsilon_n = \upsilon_0\sqrt{1+2\delta}$; and η is the anellipticity parameter given by $\eta = \frac{\varepsilon-\delta}{1+2\delta}$.

From equations (C-3) and (C-4), the magnitude of group velocity V_g is obtained,

$$V_g = \sqrt{\upsilon^2 + \left(\frac{d\upsilon}{d\theta}\right)^2}, \tag{C6}$$

and the group angle ϕ can be expressed in terms of phase velocity υ and phase angle θ,

$$\sin\phi = \frac{\upsilon\sin\theta + \frac{d\upsilon}{d\theta}\cos\theta}{\sqrt{\upsilon^2 + \left(\frac{d\upsilon}{d\theta}\right)^2}}. \tag{C7}$$

We rewrite equation (C-7) as

$$\sin\phi = \frac{\upsilon\zeta + (1-\zeta^2)\frac{d\upsilon}{d\zeta}}{\sqrt{\upsilon^2 + (1-\zeta^2)\left(\frac{d\upsilon}{d\zeta}\right)^2}}, \tag{C8}$$

where ζ denotes $\sin\theta$ for simplicity. The magnitude of group velocity V_g, phase velocity υ, and its derivative can be written in terms of ζ,

$$V_g(\zeta) = \sqrt{\upsilon^2 + (1-\zeta^2)\left(\frac{d\upsilon}{d\zeta}\right)^2}, \tag{C9}$$

$$2\upsilon^2(\zeta) = \upsilon_{p0}^2 + \left[(1+2\eta)\upsilon_n^2 - \upsilon_{p0}^2\right]\zeta^2 + \sqrt{\left[\left[(1+2\eta)\upsilon_n^2 - \upsilon_{p0}^2\right]^2 + 8\eta\upsilon_n^2\upsilon_{p0}^2\right]\zeta^4 + 2\left[(1-2\eta)\upsilon_n^2 - \upsilon_{p0}^2\right]\upsilon_{p0}^2\zeta^2 + \upsilon_{p0}^4}, \tag{C10}$$

$$\frac{d\upsilon(\zeta)}{d\zeta} = \frac{1}{2\upsilon(\zeta)}\left[(1+2\eta)\upsilon_n^2 - \upsilon_{p0}^2\right]\zeta + \frac{\left[\left[(1+2\eta)\upsilon_n^2 - \upsilon_{p0}^2\right]^2 + 8\eta\upsilon_n^2\upsilon_{p0}^2\right]\zeta^3 + \left[(1-2\eta)\upsilon_n^2 - \upsilon_{p0}^2\right]\upsilon_{p0}^2\zeta}{2\upsilon(\zeta)\sqrt{\left[\left[(1+2\eta)\upsilon_n^2 - \upsilon_{p0}^2\right]^2 + 8\eta\upsilon_n^2\upsilon_{p0}^2\right]\zeta^4 + 2\left[(1-2\eta)\upsilon_n^2 - \upsilon_{p0}^2\right]\upsilon_{p0}^2\zeta^2 + \upsilon_{p0}^4}}. \tag{C11}$$

Since $\sin\phi$ and ζ given in equation (C-8) are the limited quantities varying from 0 to 1, the value of ζ can be inverted stably from equations (C-2) and (C-8)–(C-11) by a numerical method, e.g., steepest descent approach. It follows that the reflection travel time is calculated from equations (C-1) and (C-9)–(C-11).

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