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Generalized moveout approximation for P–SV converted waves in vertically inhomogeneous transversely isotropic media with a vertical symmetry axis

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ABSTRACT

We present an overall description of moveout formulas of P–SV converted waves in vertically inhomogeneous transversely isotropic media with a vertical symmetry axis by using the generalized moveout approximation. The term "generalized" means that this approximation can be reduced to some existing approximations by specific selections of parameters, which provides flexibility in application depending on objectives. The generalized moveout approximation is separately expressed in the phase and group domains. All five parameters of the group domain (or phase domain) generalized moveout approximation are determined using the zero offset (or horizontal slowness) ray and an additional nonzero offset (or horizontal slowness) ray. We discuss the selection of parameters to link the generalized moveout approximation to some existing approximations. The approximations presented are tested on homogeneous, factorized, and layered transversely isotropic models. The results illustrate that utilizing an additional reference ray significantly improves the accuracy of phase-domain and group-domain moveout approximations for a large range of horizontal slownesses and source–receiver offsets.

Key words: Converted wave, Travel time, TI.

INTRODUCTION

The anisotropic aspect of seismic modelling and processing is now widely taken into account in applied seismology. Transverse isotropy is often observed in sedimentary rocks at long seismic wavelengths. Thomsen's (1986) notation is helpful to characterize transversely isotropic media with a vertical symmetry axis (VTI).

The interest in inverting for the vertical velocity of SV-waves and Thomsen parameters from surface seismic data in VTI media increases with the development of multi-component seismic processing and imaging in shale reservoirs (e.g., Thomsen 1999; Grechka *et al.* 2002; Cai and Tsvankin 2013). If we use only P-wave surface data, however, it is known that the SV-wave velocity cannot be estimated because the P-wave anisotropic velocity is insensitive to the vertical velocity of SV-waves (e.g., Alkhalifah 1997; Tsvankin and Thomsen 1994). In P-wave velocity analysis, only two-way zero-offset travel time, normal moveout (NMO) velocity, and the anellipticity parameter can be estimated. Compared with P-wave data, converted wave data carry more information. The P–SV converted wave is often observed in surface multi-component data from VTI media, where the term "P–SV" implies a particular conversion: a downward-propagating P-wave, converting on reflection at its deepest point of penetration to an upward-propagating SV-wave (e.g., Stewart *et al.* 2002; Thomsen 1999).

Theoretically, vertical velocities of P- and SV-waves and Thomsen's parameters (including ε and δ) can be inverted by combining the velocity analysis of P-waves and P–SV-waves and the generalized Dix formulas (Ursin and Stovas 2005; Haugen, Ursin, and Tovas 2007). The velocity analysis based

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on moveout approximations of P–SV-waves can help provide an NMO velocity model, for the next step of processing seismic data, such as Dix inversion, NMO correction, and Kirchhoff time migration. The velocity analysis of reflection data is performed in the travel-time–offset (t-x) domain (we call it the group domain for simplicity) (e.g., Alkhalifah and Tsvankin 1995; Alkhalifah 1997; Stewart *et al.* 2002; Thomsen 1999; Alkhalifah 2011; Waheed *et al.* 2013) or in the travel-time intercept-horizontal slowness (τ-p) domain (we call it the phase domain for simplicity) (e.g., van der Bann and Kendall 2002, 2003; Diebold and Stoffa 1981; Masoomzadeh *et al.* 2012; Sen and Mukherjee 2003). Despite the limitation of the velocity analysis based on the horizontal layering assumption, the interval parameters estimated from P- and P–SV-wave moveout by means of the Dix equation (Ursin and Stovas 2005) may be implemented as initial models for velocity-independent layer stripping (Dewangan and Tsvankin 2006; Tsvankin 2001; Wang and Tsvankin 2009) and reflection tomography (Foss, Ursin, and de Hoop 2005; Wang and Tsvankin 2013).

Analytic moveout formulas play a prominent role in velocity analysis. For flat-lying VTI layers, the P–SV-wave travel time of P–SV converted waves is an even function of the source–receiver offset in the group domain or of the horizontal slowness in the phase domain. Since the P–SV-wave moveout of P–SV converted waves is inherently non-hyperbolic even for a horizontal isotropic layer, non-hyperbolic moveout approximations are desirable for P–SV-wave velocity analysis. For P–SV-waves in the group domain (t-x), the rational approximation (e.g., Tsvankin and Thomsen 1994; Thomsen 1999; Li and Yuan 2003) matches the four-order moveout expansions at zero offset, and the asymptotic moveout expansions at the infinite source–receiver offset. To enhance the accuracy of rational approximation around the zero-offset ray, the sixth-order term of the P–SV-wave moveout expansion can also be introduced (Ursin and Stovas 2006). The corresponding phase-domain mirror approximation can be obtained from the group-domain approximation. An example of the phase-domain moveout approximation for P-waves in VTI media is shown in Stovas and Fomel (2012a, b).

For many P-wave moveout approximations, some authors (Zhang and Uren 2001; van der Baan and Kendall 2002; Stovas and Ursin 2004; Douma and Calvert 2006) noted the limited accuracy at large offsets. It is possible that the moveout approximations mentioned earlier become inaccurate at large offsets (horizontal slowness). To overcome this disadvantage, in addition to the zero-offset ray, three reference rays with finite offsets are introduced by Douma and Calvert (2006) to form the [2/2] Pade approximation for P-wave moveout in VTI media. Their approximation illustrates the improvement of accuracy within the range between selected rays. As an alternative, the generalized moveout approximation is proposed by Fomel and Stovas (2010), Stovas (2010a), and Stovas and Fomel (2012a). The term "generalized" means that their approximation may be reduced to many existing approximations under some particular assumptions for the selection of parameters. The generalized approximation provides a possibility for an overall description of moveout approximations. The generalized moveout approximation in the group domain includes five independent parameters, three of which are determined from the moveout expansion at the zero-offset ray and others of which are obtained from the travel time and its first-order derivative for a selected non-zero source–receiver offset reference ray. It is reasonably believed that the accuracy of the moveout approximation can be improved within the offset range between the zero-offset ray and the selected finite-offset ray. For the generalized moveout approximation in the phase-domain, all five independent parameters can be specified by a similar procedure.

In this study, we revisit the generalized moveout approximation and apply it to P–SV-waves in VTI media. Under some particular parameter selections, some simple approximations are linked to the generalized moveout approximation. The paper is organized as follows. We start with the generalized moveout approximations for P–SV-waves in the group and phase domains. In the following section, we introduce several other approximations as special cases of the generalized moveout approximation in the group and phase domains. Then, we discuss the estimation of all five parameters in the generalized moveout approximations for homogeneous VTI media, factorized VTI media, and multi-layered VTI media. In the discussion, we explain the application of the generalized moveout approximation in practice.

GENERALIZED MOVEOUT APPROXIMATION

Group-domain generalized moveout approximation

The generalized non-hyperbolic moveout approximation in the group domain is represented in the following form (Fomel and Stovas 2010):

$$t^2(x) = t_0^2 + \frac{x^2}{\upsilon_n^2} + \frac{Ax^4}{\upsilon_n^4\left(t_0^2 + B_1\frac{x^2}{\upsilon_n^2} + \sqrt{t_0^4 + 2B_1 t_0^2\frac{x^2}{\upsilon_n^2} + C_1\frac{x^4}{\upsilon_n^4}}\right)}, \tag{1}$$

where t denotes the two-way travel time; x denotes the source–receiver offset; t_0 denotes the zero-offset two-way travel time; parameters υ_n and A denote the NMO velocity and the quartic moveout coefficient, respectively; parameters t_0, υ_n, and A are obtained from the following Taylor series:

$$t^2(x) = t_0^2 + \frac{x^2}{\upsilon_n^2} + \frac{Ax^4}{2t_0^2\upsilon_n^4} + \cdots \tag{2}$$

and additional parameters B_1 and C_1 are computed from a non-zero offset reference ray with travel time $t(X) = T$ and horizontal slowness $(dt/dx)(X) = P$, i.e.,

$$B_1 = \frac{t_0^2(X - PT\upsilon_n^2)}{X(t_0^2 - T^2 + PTX)} - \frac{AX^2}{X^2 + \upsilon_n^2(t_0^2 - T^2)}, \tag{3}$$

$$C_1 = \frac{t_0^4(X - PT\upsilon_n^2)^2}{X^2(t_0^2 - T^2 + PTX)^2} + \frac{2A\upsilon_n^2 t_0^2}{X^2 + \upsilon_n^2(t_0^2 - T^2)}. \tag{4}$$

For the reference ray with an infinite offset, parameters B_1 and C_1 can be determined from the asymptotic expansion of travel time squared, i.e.,

$$B_1 = \frac{t_0^2(1 - \upsilon_n^2 P_\infty^2)}{t_0^2 - T_\infty^2} - \frac{A}{1 - \upsilon_n^2 P_\infty^2}, \tag{5}$$

$$C_1 = \frac{t_0^4(1 - \upsilon_n^2 P_\infty^2)^2}{(t_0^2 - T_\infty^2)^2}, \tag{6}$$

where T_∞ and P_∞ are the travel-time intercept and the horizontal slowness from the asymptotic expansion of travel time squared at infinite offset, i.e.,

$$t^2(x) \approx T_\infty^2 + P_\infty^2 x^2. \tag{7}$$

Phase-domain generalized moveout approximation

The phase-domain travel-time intercept τ is related to the group-domain travel time t via τ–p transform, i.e.,

$$\tau(p) = t(p) - px(p), \tag{8}$$

where the travel-time intercept τ is a function of the horizontal slowness p, and x and t denote the source–receiver offset and travel time in the group domain, respectively.

The generalized non-elliptic moveout approximation in the phase domain is represented in the form similar to the one defined in the group domain (Stovas and Fomel 2012a), i.e.,

$$\tau^2(p) = \tau_0^2\left(1 - p^2\upsilon_n^2 + \frac{Ap^4\upsilon_n^4}{1 - B_2p^2\upsilon_n^2 + \sqrt{1 - 2B_2p^2\upsilon_n^2 + C_2p^4\upsilon_n^4}}\right), \tag{9}$$

where τ_0 is the two-way zero-horizontal-slowness travel time equal to t_0 given in equation (1), parameters υ_n and A are described after equation (1), and parameters B_2 and C_2 control the accuracy of the moveout approximation (9) at large horizontal slowness.

Expanding equation (9) into a series with respect to horizontal slowness p results in the following:

$$\tau^2(p) = \tau_0^2\left(1 - p^2\upsilon_n^2 + \frac{A}{2}p^4\upsilon_n^4 + \cdots\right). \tag{10}$$

As an alternative approach, parameters υ_n and A can be specified by matching the Taylor expansion (10) and the corresponding expansion of phase-domain exact travel time squared at the zero horizontal slowness $p = 0$.

To determine parameters B_2 and C_2 given in equation (9), we select a reference ray with a non-zero-valued horizontal slowness $P \le p_{\max}$, where $p_{\max}$ denotes the maximum horizontal slowness. We define the travel-time intercept $\hat{\tau} = \tau(P)$ and its first-order derivative $\hat{\tau}' = \tau'(P)$ for the selected reference ray. From these operations, it follows that parameters B_2 and C_2 are obtained, i.e.,

$$B_2 = \frac{P\upsilon_n^2\tau_0^2 + \hat{\tau}\hat{\tau}'}{P\upsilon_n^2\left(\tau_0^2 - \hat{\tau}^2 + P\hat{\tau}\hat{\tau}'\right)} + \frac{A\tau_0^2P^2\upsilon_n^2}{\tau_0^2(1 - P^2\upsilon_n^2) - \hat{\tau}^2}, \tag{11}$$

$$C_2 = \frac{(P\upsilon_n^2\tau_0^2 + \hat{\tau}\hat{\tau}')^2}{P^2\upsilon_n^4(\tau_0^2 - \hat{\tau}^2 + P\hat{\tau}\hat{\tau}')^2} + \frac{2A\tau_0^2}{\tau_0^2(1 - P^2\upsilon_n^2) - \hat{\tau}^2}. \tag{12}$$

We next consider two special cases. For the first case, we assume $\hat{\tau} = \lim_{p\to P}\tau'(p) = \infty$ and $\hat{\tau}' = \tau(P) \neq 0$. It follows that the expressions for B_2 and C_2 are reduced to

$$B_2 = \frac{1}{P^2\upsilon_n^2} + \frac{A\tau_0^2P^2\upsilon_n^2}{\tau_0^2(1 - P^2\upsilon_n^2) - \hat{\tau}^2}, \tag{13}$$

$$C_2 = \frac{1}{P^4\upsilon_n^4} + \frac{2A\tau_0^2}{\tau_0^2(1 - P^2\upsilon_n^2) - \hat{\tau}^2}. \tag{14}$$

For the second case, it is assumed that $\hat{\tau}' = \lim_{p\to P}\tau'(p) = \infty$ and $\hat{\tau} = \tau(P) = 0$. From the limit $\sigma = \lim_{p\to P}[\tau'(p)\tau(p)]$, it follows that equations (13) and (14) become

$$B_2 = \frac{P\upsilon_n^2\tau_0^2 + \sigma}{P\upsilon_n^2(\tau_0^2 + P\sigma)} + \frac{A\tau_0^2P^2\upsilon_n^2}{\tau_0^2(1 - P^2\upsilon_n^2)}, \tag{15}$$

$$C_2 = \frac{(P\upsilon_n^2\tau_0^2 + \sigma)^2}{P^2\upsilon_n^4(\tau_0^2 + P\sigma)^2} + \frac{2A\tau_0^2}{\tau_0^2(1 - P^2\upsilon_n^2)}. \tag{16}$$

OTHER MOVEOUT APPROXIMATIONS

Fourth-order Taylor approximation

By setting $B = C = 0$, the generalized moveout approximation (1) is reduced to the fourth-order Taylor approximation in the group domain, i.e.,

$$t^2(x) = t_0^2 + \frac{x^2}{\upsilon_n^2} + \frac{Ax^4}{2t_0^2\upsilon_n^4}. \tag{17}$$

Similarly, the generalized moveout approximation (9) becomes the fourth-order Taylor approximation in the phase domain, i.e.,

$$\tau^2(p) = \tau_0^2\left(1 - p^2\upsilon_n^2 + \frac{A}{2}p^4\upsilon_n^4\right). \tag{18}$$

Shifted hyperbola and ellipse approximations

The selection of parameters $A = (1 - s)/2$, $B = A + 1/2$, and $C = 0$ reduces the generalized moveout approximations (1) and (9) to the shifted hyperbola and ellipse approximations in the group and phase domains. The shifted hyperbola approximation is given by

$$t(x) = t_0\left(1 - \frac{1}{s}\right) + \frac{1}{s}\sqrt{t_0^2 + s\frac{x^2}{\upsilon_n^2}}. \tag{19}$$

The corresponding shifted ellipse approximation in the phase domain is the mirror of the shifted hyperbola approximation (Stovas and Fomel 2012b), i.e.,

$$\tau(p) = \tau_0\left[\left(1 - \frac{1}{s}\right) + \frac{1}{s}\sqrt{1 - sp^2\upsilon_n^2}\right]. \tag{20}$$

The heterogeneity coefficient s controls the deviation of equation (19) from the hyperbola and the deviation of equation (20) from the ellipse (Stovas and Fomel 2012b).

Rational approximations

By setting $C_1 = B_1^2 \equiv D$, the generalized moveout approximation (1) is reduced to the rational moveout approximation in the group domain:

$$t^2(x) = t_0^2 + \frac{x^2}{\upsilon_n^2} + \frac{Ax^4}{2t_0^2\upsilon_n^4\left(1 + \frac{Dx^2}{t_0^2\upsilon_n^2}\right)}. \tag{21}$$

Similarly, the corresponding rational approximation in the phase domain is obtained by setting $C_2 = B_2^2 \equiv D$ in generalized moveout approximation (9), i.e.,

$$\tau^2(p) = \tau_0^2\left(1 - p^2\upsilon_n^2 + \frac{Ap^4\upsilon_n^4}{2(1 - Dp^2\upsilon_n^2)}\right). \tag{22}$$

Thomsen (1999) suggested determining the parameter D by matching the asymptotic expansions of the exact travel time squared and the moveout approximation (21) at the infinite offset. Thus, parameter D is given by

$$D = \frac{A\upsilon_{pn}^2(1 + 2\eta)}{2(\upsilon_n^2 - \upsilon_{pn}^2(1 + 2\eta))}, \tag{23}$$

where υ_{pn} and η represent the P-wave NMO velocity and the anellipticity parameter (Alkhalifah and Tsvankin 1995), which can be obtained from the Taylor expansion of P-wave travel time squared at the zero-offset ray.

Ursin and Stovas (2006) proposed an alternative way to determine parameter D by matching the Taylor expansion of rational approximation (21) and the exact travel time squared at the zero-offset ray up to the sixth order, i.e.,

$$D = -\frac{E}{3A}, \tag{24}$$

where parameter E is related to the sixth-order moveout coefficient. For a homogeneous vertical transverse isotropy (VTI) medium, parameter E is given by equation (D-27) of Appendix D. For a factorized VTI medium (e.g., Alkhalifah 1995; Stovas 2010b), see equation (D-23) with equations (D-15)–(D-18) in Appendix D. For a multi-layered VTI model, see equation (E-13) in Appendix E.

NUMERICAL TESTS

In this section, we apply the generalized moveout approximation to specific models including a homogeneous vertical transverse isotropy (VTI) model, a factorized VTI model, and a multi-layered VTI model. To quantitatively compare the accuracy of the approximations, we slightly modify the two-point P-wave ray tracing algorithm (Tian and Chen 2005; Fowler *et al.* 2008) to calculate the exact P–SV-wave travel time in the group domain. The exact P–SV-wave travel time in the phase domain can be calculated by using the vertical slowness formulas of P- and SV-waves (van der Baan and Kendall 2003). The following notation is used to describe VTI media: α_0 denotes P-wave vertical velocity; r_0 denotes the ratio between the vertical S-wave to P-wave velocities; and $a_0 = 2\delta$ and $b_0 = 2(\varepsilon - \delta)/r_0^2$ denote the P-wave and SV-wave NMO factors, respectively, where ε and δ are Thomsen (1986) parameters.

A homogeneous VTI model

For a horizontal reflector in a homogeneous VTI medium, we derive the analytical expressions for all parameters given in the group- and phase-domain generalized moveout approximations (1) and (9). By expanding the exact P–SV-wave travel time (see Appendix A) squared at zero offset, we determine the zero-offset two-way travel time t_0, NMO velocity υ_n, and coefficient A (see Appendix D), i.e.,

$$t_0 = t_{p0} + t_{s0} = t_{p0}\left(1 + \frac{1}{r_0}\right), \tag{25}$$

$$\upsilon_n^2 = \alpha_0^2 \frac{r_0(1 + a_0 + (1 + b_0)r_0)}{1 + r_0}, \tag{26}$$

$$A = -\frac{(1 + a_0 + (-1 + b_0)r_0^2)^2}{2r_0(1 + a_0 + (1 + b_0)r_0)^2}. \tag{27}$$

Here, t_{p0} and t_{s0} given in equation (25) denote P- and SV-wave one-way zero-offset travel times.

To obtain the analytical expression for parameters B_1 and C_1 given in group-domain generalized moveout approximation (1) and parameters B_2 and C_2 given in phase-domain generalized moveout approximation (9), we consider the limit case that the incident branch of the selected reference ray becomes horizontal.

For the group-domain generalized moveout approximation, this limit case indicates that the source–receiver offset of the reference ray is infinite. From equations (3) and (4), it follows that parameters B_1 and C_1 are given by (see Appendix B)

$$B_1 = \frac{(1 + r_0)\left(1 + a_0 + (-1 + b_0)\, r_0{}^2\right)^2 \left(1 + a_0 + b_0 r_0{}^2\right)}{2r_0(1 + a_0 + (1 + b_0)\, r_0)^2 \left(1 + a_0 + (-1 + b_0 r_0)\, r_0^2\right)}, \tag{28}$$

$$C_1 = 0. \tag{29}$$

For the phase-domain generalized moveout approximation, this limit case indicates that the horizontal slowness of the reference ray become the maximum horizontal slowness for the incident P-wave. From equations (13) and (14), it follows that parameters B_2 and C_2 are given by (see Appendix C)

$$B_2 = \frac{(1 + r_0)(1 + a_0 + (1 + b_0)r_0^2)}{2r_0(1 + a_0 + (1 + b_0)r_0)}, \tag{30}$$

$$C_2 = \frac{(1 + r_0)^2(1 + a_0 + b_0 r_0^2)}{(1 + a_0 + (1 + b_0)r_0)^2}. \tag{31}$$

We design a homogeneous VTI model to test the generalized moveout approximations in the group and phase domains. The model parameters are P-wave vertical velocity $\alpha_0 = 3$km/s, the ratio between the vertical velocities of SV- and P-waves $r_0 = 0.5$, Thomsen parameters $\varepsilon = 0.1$ and $\delta = -0.1$, and the reflector depth $z = 1$km. The plot (a) in Fig. 1 compares the group-domain generalized moveout approximation with other approximations. This plot shows that the generalized moveout approximation with the horizontal reference ray can produce a satisfactory result, although it has relatively low accuracy at large offset compared with the rational approximation (21) with equation (24); for a selection of the large-offset reference ray, the accuracy of the generalized moveout approximation can be significantly improved.

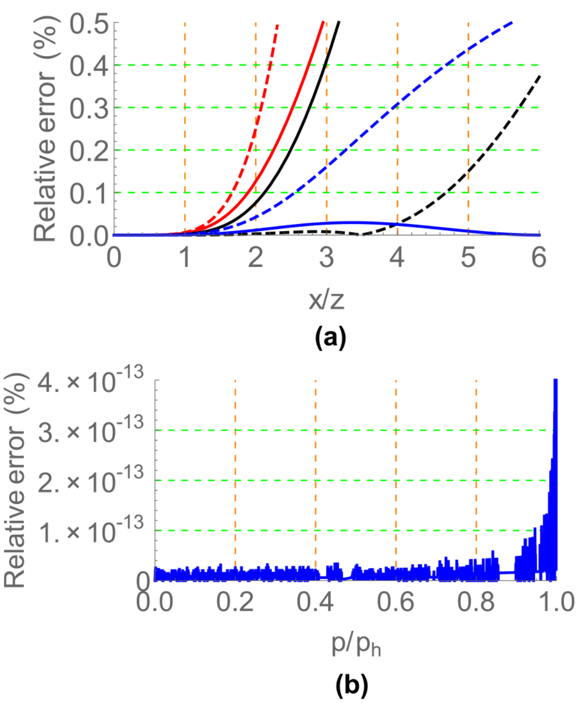


Figure 1 Relative error of the P–SV-wave moveout approximations in the (a) group domain and the (b) phase domain for a homogeneous VTI layer. The plot (a) compares the relative error of fourth-order Taylor approximation (17) (red dashed line), shifted-hyperbola approximation (19) (red line), rational approximation (21) with equation (23) (black line), rational approximation (21) with equation (24) (black dashed line), the generalized moveout approximation (1) with the horizontal reference ray (blue dashed line), and the generalized moveout approximation (1) with a finite-offset (6km) reference ray (blue line), respectively. The lateral coordinate x/z denotes the ratio of source–receiver offset to reflector depth. The plot (b) illustrates the relative error of the phase-domain generalized moveout approximation (9) with the horizontal reference ray. p_h denotes the maximum horizontal slowness of P-waves in this model.

The plot in (b) Fig. 1 shows the relative error of the phase-domain generalized moveout approximation. The relative error in travel time is small enough, which seems that the phase-domain generalized moveout approximation is equivalent to the exact solution.

A horizontal reflector in a factorized VTI medium

For a horizontal reflector in a factorized VTI medium, P- and SV-wave vertical velocities are linear functions of depth z. It is assumed that the ratio between P- and SV-wave vertical velocities keeps a constant for simplicity. Hence, the vertical velocities for P- and SV-waves are represented by

$$\alpha(z) = \alpha_0 \left(1 + \frac{\gamma - 1}{H} z\right), \tag{32}$$

$$\beta(z) = r_0 \alpha_0 \left(1 + \frac{\gamma - 1}{H} z\right), \tag{33}$$

where $\alpha(z)$ and $\beta(z)$ denote P- and SV-waves vertical velocities at the depth z, respectively; α_0 denote P-wave vertically velocity at surface; H is the thickness of this factorized VTI layer; and $\gamma = \alpha(H)/\alpha_0$ is the ratio between the vertical velocity of P-wave to the bottom and the vertical velocity to the top of the layer.

For P–SV-waves in the factorized VTI layer, we finally derive the zero-offset two-way travel time t_0, NMO velocity υ_n, and coefficient A as follows (see Appendix D):

$$t_0 = \frac{H(1 + r_0)\ln\gamma}{r_0 \alpha_0 (\gamma - 1)}, \tag{34}$$

$$\upsilon_n^2 = \alpha_0^2 \frac{r_0(1 + a_0 + (1 + b_0) r_0)(\gamma^2 - 1)}{2(1 + r_0)\ln\gamma}, \tag{35}$$

$$A = \frac{1}{2} - \frac{((1 + a_0)^2(1 + r_0) + 4(1 + a_0) b_0 r_0^2 + (1 + b_0)^2 r_0^3 + (-1 + b_0)^2 r_0^4)(1 + \gamma^2)\ln\gamma}{2 r_0 (1 + a_0 + (1 + b_0) r_0)^2 (\gamma^2 - 1)}. \tag{36}$$

By setting $\gamma = 1$, we obtain equations (25)–(27). By setting $a_0 = b_0 = 0$, we obtain equations for the linear velocity model (Stovas 2010b).

Parameters B_1 and C_1 given in the group-domain approximation (1) can be determined by substituting the offset $X(P)$ and travel time $T(P)$ for the maximum-offset reference ray with the horizontal slowness $P = 1/(\alpha_0\gamma)$ into equations (3) and (4), where the travel time $T(P)$ and the offset $X(P)$ are calculated by the parametric equations (D-3) and (D-5) in Appendix D. On the other hand, parameters B_2 and C_2 given in the phase-domain generalized moveout approximation (9) can be calculated by substituting travel-time intercept $\hat{\tau} = \tau(P)$ and its derivative $\hat{\tau}' = \tau'(P)$ for the maximum-offset reference ray with the horizontal slowness $P = 1/(\alpha_0\gamma)$ into equations (11) and (12), where $\hat{\tau}$ is obtained from τ–p transform (8) and $\hat{\tau}'$ is obtained by differentiating $\tau(P)$ with respect to horizontal slowness P at $P = 1/(\alpha_0\gamma)$.

We use a factorized VTI model to compare the generalized moveout approximations in the group and phase domains with other approximations. The medium parameters are $\alpha_0 = 3$km/s, $r_0 = 0.5$, $\gamma = 1.5$, $\varepsilon = 0.1$, and $\delta = -0.1$. The thickness of this horizontal layer is 1 km. Figure 2 shows the relative error of the generalized moveout approximations and other approximations. The plot (a) in Fig. 2 illustrates that the group-domain generalized moveout approximation is almost close to the exact solution, and its maximum relative error is less than 0.02%. The reflected P–SV-wave with the maximum offset is taken as the non-zero offset reference ray for the group generalized moveout approximation. This corresponds to selecting the maximum horizontal slowness of reflected P–SV-waves for the phase-domain generalized moveout approximation. The plot (b) in Fig. 2 shows that the phase-domain generalized moveout approximation also produces a relatively accurate result compared with other approximations, especially at very large horizontal slowness. Except the generalized moveout approximation, we note that the accuracy of other phase-domain approximations decreases rapidly when the horizontal slowness of P–SV-waves approaches its maximum value.

Multi-layered VTI model

For P–SV-waves in a multi-layered VTI model, the moveout expansion at the zero-offset ray is given as follows (see Appendix E):

$$t_0 = d_0, \tag{37}$$

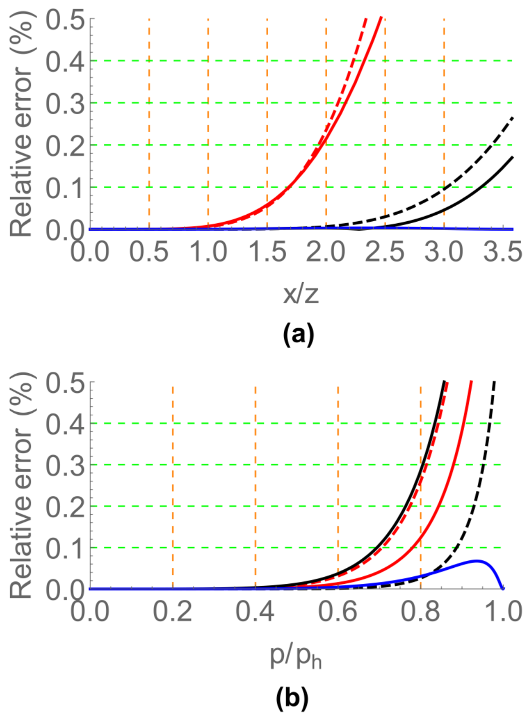


Figure 2 Relative error of the P–SV moveout approximations for the (a) group domain and the (b) phase domain in a factorized VTI model. The plot (a) compares the relative error of fourth-order Taylor approximation (17) (red line), shifted-hyperbola approximation (19) (red dashed line), rational approximation (21) with equation (23) (black line), rational approximation (21) with equation (24) (black dashed line), and the generalized moveout approximation (1) with the maximum offset reference ray (blue line) versus the offset–depth ratio x/z. The plot (b) illustrates the relative error of the phase-domain approximations versus the normalized horizontal slowness p/p_h, which includes fourth-order Taylor approximation (18) (red line), shifted-hyperbola approximation (20) (red dashed line), rational approximation (22) with equation (23) (black line), rational approximation (22) with equation (24) (black dashed line), and the generalized moveout approximation (1) with the normalized horizontal slowness of the reference ray (blue line) equal to 0.99. p_h denotes the maximum horizontal slowness of P-waves in this model.

$$\upsilon_n = \sqrt{-\frac{2d_2}{d_0}}, \tag{38}$$

$$A = \frac{1}{2} + \frac{d_0 d_4}{d_2^2}. \tag{39}$$

Here, $d_i, i = 0, 2, 4$, are calculated from equations (E-4)–(E-7) in Appendix E.

For the group-domain generalized moveout approximation (1), a finite-offset reference ray is selected to determine parameters B_1 and C_1 from equations (3) and (4).

Table 1 Parameters for a six-layer VTI model. Δz denotes the layer thickness; α_0 and β_0 are the vertical velocities of P- and SV-waves, respectively; and ε and δ are Thomsen (1986) parameters. The values of these parameters are taken from Ursin and Stovas (2006)

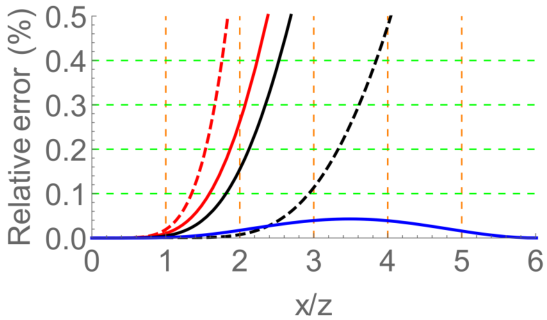
Layer	Δz (km)	α_0(km/s)	β_0(km/s)	ε	δ
1	0.25	1.74	0.39	0.08	0.05
2	0.15	1.85	0.62	0.14	0.10
3	0.10	1.94	0.78	0.10	0.03
4	0.16	2.14	0.86	0.14	−0.02
5	0.14	2.22	0.89	0.10	−0.05
6	0.20	2.00	1.00	0.14	0.10

For the phase-domain generalized moveout approximation, parameters B_2 and C_2 are determined from equations (11) and (12).

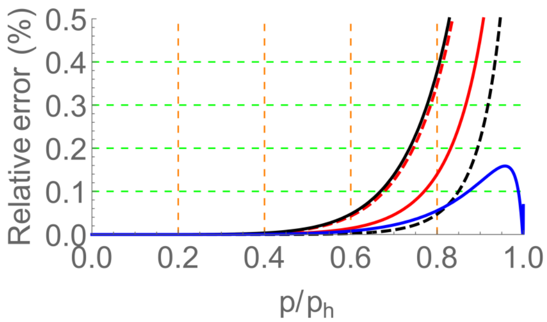
A six-layer VTI model is designed to calculate the error of the generalized moveout approximation and other existing approximations in the group and phase domains. The model parameters are listed in Table 1. Figure 3 shows comparisons of the generalized moveout approximation with other approximations for this model. The plot in Fig. 3 shows that the group-domain generalized moveout approximation may provide a satisfactory accuracy for multi-layered media by using the large-offset reference ray. For the phase-domain generalized moveout approximation in the right plot of Fig. 3, we select the reference ray with the normalized horizontal slowness equal to 0.99. This means that the selected ray is very close to the horizontal ray. Similar to the previous example of a factorized VTI medium, the phase-domain generalized moveout approximation produces accurate results by comparing with other approximations.

DISCUSSION

To apply the group-domain generalized moveout approximation in velocity analysis, we select a reference ray corresponding to a large source–receiver offset along a specific seismic event in a common midpoint (CMP) gather. This may be accomplished when the signal-to-noise ratio of seismic data is high. The local slope of the travel-time–offset curve for the selected ray may be automatically estimated for example using the plane-wave deconstruction approach (e.g., Fomel 2002; Schleicher *et al.* 2009). In the generalized moveout approximation (1) with equations (3) and (4), only zero-offset travel time, NMO velocity, and the quartic moveout coefficients remain unknown. The three quantities can be estimated by the double semblance scanning (e.g., Li and Yuan 2003). From



(a)



(b)

Figure 3 Relative error of the P–SV-wave moveout approximations for the (a) group domain and the (b) phase-domain in a six-layer VTI model. Model parameters are listed in Table 1. The plot (a) shows the relative error of fourth-order Taylor approximation (17) (red line), shifted-hyperbola approximation (19) (red dashed line), rational approximation (21) with equation (23) (black line), rational approximation (21) with equation (24) (black dashed line), and the generalized moveout approximation (1) with a finite-offset (6 km) reference ray (blue line) versus the offset–depth ratio x/z. The plot (b) illustrates the relative error of the phase-domain approximations versus the normalized horizontal slowness p/p_h, which includes fourth-order Taylor approximation (18) (red line), shifted-hyperbola approximation (20) (red dashed line), rational approximation (22) with equation (23) (black line), rational approximation (22) with equation (24) (black dashed line), and the generalized moveout approximation (1) with the maximum horizontal slowness reference ray (blue line). In the plot (b), p_h denotes the maximum horizontal slowness of P-waves in this model.

the accuracy analysis in the section of "Numerical examples," it is reasonable to believe that the estimated NMO velocity and the quartic moveout coefficients are accurate enough for the normal moveout correction in horizontally vertical transverse isotropy (VTI) layers. When using the generalized moveout approximation to do the NMO correction of the CMP data, the effect of stretching of data may be reduced since a long-offset reference ray is involved to constrain the generalized moveout approximation. A similar phenomenon is noted by Douma and Calvert (2006), who used four pairs of travel time and offset of rays to build an accurate rational moveout approximation and found that their approximation can reduce the stretching of long offset data during NMO correction. In practice, it is obvious that the accuracy of the generalized moveout approximation is limited by the accuracy of the travel time and the slope of the selected reference ray. When the quality of the CMP data is too low to select the long-offset ray, the selection of a nonzero-offset reference ray does not help moveout approximation very much. For this case, therefore, the generalized moveout approximation needs to be replaced by relatively simple moveout approximations (e.g., rational approximation or shifted hyperbola approximation) in velocity analysis.

The phase-domain velocity analysis may be done in a similar way. For the phase-domain generalized moveout approximation, we emphasize that the horizontal slowness of the selected reference ray must be close to the maximum horizontal slowness. The maximum horizontal slowness corresponds to the intersection of two phase-domain seismic travel-time curves coming from the upper and lower interfaces of a horizontal layer. The third numerical example in the previous section shows that, for horizontally layered VTI media, all phase-domain approximations, except the generalized moveout approximation, suffer the problem of rapid increase in error when the horizontal slowness tends to the maximum. This indicates that the generalized moveout approximation helps reduce "data stretching" during the phase-domain NMO correction. On the other hand, the "layer stripping" velocity analysis is easier in the phase domain than in the group domain since the horizontal slowness of a considered ray is preserved when passing through horizontal layers. The first numerical example in the previous section shows that when selecting the reference ray with the maximum horizontal slowness, the phase-domain generalized moveout approximation is equivalent to the exact solution. It means that the phase-domain generalized moveout approximation can be used to invert medium parameters for a single horizontal VTI layer without any loss of accuracy. For horizontally VTI media, this approximation may be used in the layer-stripping approach.

To further obtain the interval vertical velocities and Thomsen parameters, NMO velocities and quartic moveout coefficients of pure P-waves and P–SV-waves are combined by the generalized Dix-type formula for VTI media (Ursin and Stovas 2005). This is based on the assumption that both P-waves and P–SV-waves penetrate the same depth.

CONCLUSIONS

The generalized moveout approximation is presented in the phase and group domains. The generalized moveout approximation uses five parameters to describe travel times of P–SV converted waves in vertically inhomogeneous VTI media. A few relatively simple approximations are derived from the generalized moveout approximation by certain selections of parameters. The accuracy comparison with other existing approximations for VTI models shows that the generalized moveout approximation can replace the exact solution to provide accurate travel times of P–SV converted waves within a wide range of source–receiver offsets and horizontal slownesses for horizontally layered VTI media.

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REFERENCES

Alkhalifah T. 1995. Efficient synthetic-seismogram generation in transversely isotropic, inhomogeneous media. *Geophysics* **60**, 1139–1150.

Alkhalifah T. 1997. Velocity analysis using nonhyperbolic moveout in transversely isotropic media. *Geophysics* **62**, 1839–1854.

Alkhalifah T. 2011. Traveltime approximations for transversely isotropic media with an inhomogeneous background. *Geophysics* **76**, WA31–42.

Alkhalifah T. and Tsvankin I. 1995. Velocity analysis for transversely isotropic media. *Geophysics* **60**, 1550–1566.

van der Bann M. and Kendall J.-M. 2002. Estimating anisotropy parameters and traveltimes in the τ;-p domain. *Geophysics* **67**, 1076–1086.

van der Bann M. and Kendall J.-M. 2003. Traveltime and conversion-point computations and parameter estimation in layered, anisotropic media by τ-p transform. *Geophysics* **68**, 210–224.

Cai P. and Tsvankin I. 2013. Joint migration velocity analysis of PP- and PS-waves for VTI media. *Geophysics* **78**, WC123–WC135.

Dewangan P. and Tsvankin I. 2006. Velocity-independent layer stripping of PP and PS reflection traveltimes. *Geophysics* **71**, U59–U65.

Diebold J.B. and Stoffa P.L. 1981. The traveltime equation, tau-p mapping, and inversion of common midpoint data. *Geophysics* **46**, 238–254.

Douma H. and Calvert A. 2006. Nonhyperbolic moveout analysis in VTI media using rational interpolation. *Geophysics* **71**, D59–D71.

Fomel S. 2002. Applications of plane-wave destruction filters. *Geophysics* **67**, 1946–1960.

Fomel S. and Stovas A. 2010. Generalized nonhyperbolic moveout approximation. *Geophysics* **75**, U9–U18.

Foss S.K., Ursin B. and de Hoop M.C. 2005. Depth-consistent reflection tomography using PP and PS seismic data. *Geophysics* **70**, U51–U65.

Fowler P.J., Jackson A., Gaffney J. and Boreham D. 2008. Direct nonlinear traveltime inversion in layered VTI media. 78th SEG meeting, Las Vegas, USA, Expanded Abstract, 3028–3032.

Grechka V., Tsvankin I., Bakulin A., Hansen J.O. and Signer C. 2002. Joint inversion of PP and PS reflection data for VTI media: A North Sea case study. *Geophysics* **67**, 1382–1395.

Haugen J.A., Ursin B. and Stovas A. 2007. Sensitivity of Dix-type inverse formulae. *Journal of Geophysics and Engineering* **4**, 404–414.

Li X.-Y. and Yuan J. 2003. Converted-wave moveout and conversion-point equations in layered VTI media: theory and applications. *Journal of Applied Geophysics* **54**, 297–318.

Masoomzadeh H., Singh S.C. and Barton P.J. 2012. Shifted-elliptical nonstretch moveout correction of wide-angle seismic data in the τ-p domain, using an example from the Faeroe-Shetland Basin. *Geophysics* **77**, B227–B236.

Schleicher J., Costa J.C., Santos L.T., Novais A. and Tygel M. 2009. On the estimation of local slopes. *Geophysics* **74**, P25–P33.

Sen M. and Mukherjee A. 2003. τ-p analysis in transversely isotropic media. *Geophysical Journal International* **154**, 647–658.

Stewart R.R., Gaiser J.E., Brown R.J. and Lawton D.C. 2002. Converted-wave seismic exploration: Methods. *Geophysics* **67**, 1348–1363.

Stovas A. 2010a. Generalized moveout approximation for qP- and qSV waves in a homogeneous transversely isotropic medium. *Geophysics* **75**, D79–D84.

Stovas A. 2010b. Kinematical characteristics of the factorized velocity model. *Geophysical Prospecting* **58**, 219–227.

Stovas A. and Ursin B. 2004. New travel-time approximations for a transversely isotropic medium. *Journal of Geophysics and Engineering* **1**, 128–133.

Stovas A. and Fomel S. 2012a. Generalized nonhyperbolic moveout apprimation in τ;-p domain. *Geophysics* **77**, U23–U30.

Stovas A. and Fomel S. 2012b. Shifted hyperbola moveout approximation revisited. *Geophysical Prospecting* **60**, 395–399.

Thomsen L. 1986. Weak elastic anisotropy. *Geophysics* **51**, 1954–1966.

Thomsen L. 1999. Converted-wave reflection seismology over inhomogeneous anisotropic media. *Geophysics* **64**, 678–690.

Tian Y. and Chen X. 2005. A rapid and accurate two-point ray tracing method in horizontally layered velocity model. *Acta Seismologica Sinica* **18**, 154–161.

Tsvankin I. and Thomsen L. 1994. Nonhyperbolic reflection moveout in anisotropic media. *Geophysics* **59**, 1290–1304.

Tsvankin I. 2001. *Seismic Signatures and Analysis of Reflection Data in Anisotropic Media*, 1st edn. Elsevier Publishing Company, Inc.

Ursin B. and Stovas A. 2005. Generalized Dix equations for a layered transversely isotropic medium. *Geophysics* **70**, D77–D81.

Ursin B. and Stovas A. 2006. Traveltime approximations for a layered transversely isotropic medium. *Geophysics* 71, D23–D33.

Wang X. and Tsvankin I. 2009. Estimation of interval anisotropy parameters using velocity-independent layer stripping. *Geophysics* 74, WB117–WB127.

Wang X. and Tsvankin I. 2013. Multiparameter TTI tomography of P-wave reflection and VSP data. *Geophysics* 78, WC51–WC63.

Waheed U., Alkhalifah T. and Stovas A. 2013. Diffraction traveltime approximations for TI media with an inhomogeneous background. *Geophysics* 78, C139–C147.

Zhang F. and Uren N. 2001. Approximate explicit ray velocity functions and travel times for P-waves in TI media. 71st SEG meeting, San Antonio, USA, Expanded Abstracts, 106–109.

APPENDIX A

PARAMETRIC EQUATIONS FOR THE P–SV-WAVE TRAVEL TIME IN A HOMOGENEOUS VTI LAYER

In a homogeneous vertical transverse isotropy (VTI) medium, P- and SV-waves can be characterized by the vertical velocities α_0 and β_0 for P- and SV-waves and two Thomsen (1986) parameters ε and δ. To represent the reflection travel time of P–SV-waves, we adopt Stovas' (2010a) notation of travel-time parameters for P- and SV-waves in a horizontal VTI layer: P-wave vertical velocity α_0; the ratio between vertical SV- and P-wave velocities $r_0 = \beta_0/\alpha_0$; P- and SV-wave one-way vertical travel times $t_{P0} = z/\alpha_0$ and $t_{S0} = t_{P0}/r_0$ with z being the layer thickness; and P- and SV-waves NMO factors $a_0 = 2\delta$ and $b_0 = 2(\varepsilon - \delta)/r_0^2$.

The vertical slowness for P- and SV-waves in a VTI medium can be written as (Ursin and Stovas 2006)

$$q_{P(S)}^2(p) = -\frac{1}{2}\left[-q_{\alpha 0}^2(p) - q_{\beta 0}^2(p) + p^2(a_0+b_0) \pm \sqrt{\left(q_{\alpha 0}^2(p) - q_{\beta 0}^2(p)\right)^2 - 2p^2(r_0^2-1)(b_0-a_0)/(r_0^2\alpha_0^2) + p^4\left(4b_0(1-r_0^2) + (a_0+b_0)^2\right)}\right], \quad \text{(A-1)}$$

where subscripts "P" and "S" denote P- and SV-waves, respectively; the signs "+" and "-" in front of the square root correspond to P- and SV-waves, respectively; the expressions for $q_{\alpha 0}(p)$ and $q_{\beta 0}(p)$ are given by

$$q_{\alpha 0}^2(p) = 1/\alpha_0^2 - p^2, \quad \text{(A-2)}$$

$$q_{\beta 0}^2(p) = 1/(r_0^2\alpha_0^2) - p^2. \quad \text{(A-3)}$$

For a horizontal and homogeneous VTI layer, the P–SV-wave travel time t is represented in terms of horizontal slowness p as follows

$$t(p) = xp + z(q_P(p) + q_S(p)). \quad \text{(A-4)}$$

Here, slowness projections p, q_P, and q_S satisfy equation (A-1) and the following equation:

$$\frac{dq_P}{dp} + \frac{dq_S}{dp} = -\frac{x}{z}. \quad \text{(A-5)}$$

For a horizontal and homogeneous VTI layer, the source–receiver offset x and travel time t of P–SV-waves are represented in terms of horizontal slowness p as follows:

$$x(p) = x_P(p) + x_S(p), \quad \text{(A-6)}$$

$$t(p) = t_P(p) + t_S(p), \quad \text{(A-7)}$$

where subscripts "P" and "S" denote P- and SV-waves, respectively; the source–receiver offset and travel time formulations are given by (Stovas 2010a)

$$x_P(p) = p\alpha_0^2 t_{P0}\frac{1 + a_0 + S(p) + H(p)}{\sqrt{1 - p^2\alpha_0^2(1 + a_0 + S(p))}}, \quad \text{(A-8)}$$

$$t_P(p) = t_{P0}\frac{1 + p^2\alpha_0^2 H(p)}{\sqrt{1 - p^2\alpha_0^2(1 + a_0 + S(p))}}, \quad \text{(A-9)}$$

respectively, for the incident P-wave; and

$$x_S(p) = p\alpha_0^2 r_0 t_{P0}\frac{1 + b_0 - S(p) - H(p)}{\sqrt{1 - p^2\alpha_0^2 r_0^2(1 + b_0 - S(p))}}, \quad \text{(A-10)}$$

$$t_S(p) = t_{P0}\frac{1 - p^2\alpha_0^2 r_0^2 H(p)}{r_0\sqrt{1 - p^2\alpha_0^2 r_0^2(1 + b_0 - S(p))}}, \quad \text{(A-11)}$$

respectively, for the reflected SV-wave. Here, functions $H(p)$, $S(p)$, and $Q(p)$ are given by

$$H(p) = \frac{S(p)}{\sqrt{Q(p)}}, \quad \text{(A-12)}$$

$$S(p) = \frac{2b_0 p^2 \alpha_0^2 r_0^2 (1 - r_0^2 + a_0)}{(1 - r_0^2)\left(1 + \frac{(a_0 - b_0)}{(1 - r_0^2)} p^2 \alpha_0^2 r_0^2 + \sqrt{Q(p)}\right)}, \quad \text{(A-13)}$$

$$Q(p) = 1 + \frac{2(a_0 - b_0)}{(1 - r_0^2)} p^2 \alpha_0^2 r_0^2 + \frac{(4(1 - r_0^2)b_0 + (a_0 + b_0)^2)}{(1 - r_0^2)^2} p^4 \alpha_0^4 r_0^4. \quad \text{(A-14)}$$

APPENDIX B

ANALYTICAL EXPRESSIONS FOR B_1 AND C_1

In this Appendix, we derive the parameters B_1 and C_1 given in group-domain generalized moveout approximation (1) for P–SV-waves in a homogeneous vertical transverse isotropy (VTI) layer. For the horizontal reference ray, the expressions for B_1 and C_1 given by equations (3) and (4) reduce to

$$B_1 = \frac{t_0^2 (1 - \upsilon_n^2 P_\infty^2)}{t_0^2 - T_\infty^2} - \frac{A}{1 - \upsilon_n^2 P_\infty^2}, \quad \text{(B-1)}$$

$$C_1 = \frac{t_0^4 (1 - \upsilon_n^2 P_\infty^2)^2}{(t_0^2 - T_\infty^2)^2}, \quad \text{(B-2)}$$

where parameters T_∞ and P_∞ are determined from the asymptotic expansion of travel time squared at infinite offset, i.e.,

$$P_\infty = \lim_{x \to \infty} \left(\frac{dt(x)}{dx} \right), \quad \text{(B-3)}$$

$$T_\infty^2 = \lim_{x \to \infty} \left(t^2(x) - \frac{dt(x)}{dx} t(x) x \right). \quad \text{(B-4)}$$

Equations (B-3) and (B-4) follow from equation (7).

Parameter P_∞ given by equation (B-3) denotes the horizontal slowness of the P–SV-wave ray with an infinite offset. Parameter P_∞ is identical to the maximum horizontal slowness p_h of P-waves in the VTI medium, i.e.,

$$P_\infty = p_h = \frac{1}{\alpha_0 \sqrt{1 + a_0 + b_0 r_0^2}}. \quad \text{(B-5)}$$

We rewrite the term inside the bracket in equation (B-4) as

$$\begin{aligned} t^2 - \frac{dt}{dx} tx &= (t_P + t_S)^2 - p(t_P + t_S)(x_P + x_S) \\ &= (t_P^2 - p t_P x_P) + (t_S^2 - p t_S x_S) \\ &\quad + (2 t_P t_S - p t_P x_S - p t_S x_P). \end{aligned} \quad \text{(B-6)}$$

By substituting equations (A-8)-(A-11) into equation (B-6), we derive

$$\begin{aligned} t^2 - \frac{dt}{dx} tx = t_{P0}^2 \left(1 + \frac{1}{r_0^2}\right) &+ \frac{t_{P0}^2 (1 + p^2 \alpha_0^2 H(p)) \sqrt{1 - r_0^2 p^2 \alpha_0^2 (1 + b_0 - S(p))}}{r_0 \sqrt{1 - p^2 (1 + a_0 + S(p)) \alpha_0^2}} \\ &+ \frac{t_{P0}^2 (1 - r_0^2 p^2 \alpha_0^2 H(p)) \sqrt{1 - p^2 (1 + a_0 + S(p)) \alpha_0^2}}{r_0 \sqrt{1 - r_0^2 p^2 \alpha_0^2 (1 + b_0 - S(p))}}. \end{aligned} \quad \text{(B-7)}$$

For the special case of the horizontal reference ray, the horizontal slowness p becomes P_∞ given by equation (B-5). From equations (B-4) and (B-7), it follows that the asymptotic traveltime intercept T_∞ is given by

$$T_\infty^2 = \infty. \quad \text{(B-8)}$$

Substitution of equations (25)-(27) and equations (B-5) and (B-8) into equations (B-1) and (B-2) leads to the expressions for B_1 and C_1, i.e.,

$$B_1 = \frac{(1 + r_0)(1 + a_0 + (-1 + b_0) r_0^2)^2 (1 + a_0 + b_0 r_0^2)}{2 r_0 (1 + a_0 + (1 + b_0) r_0)^2 (1 + a_0 + (-1 + b_0 r_0) r_0^2)}, \quad \text{(B-9)}$$

$$C_1 = 0. \quad \text{(B-10)}$$

For the special case of an isotropic medium ($a_0 = b_0 = 0$), equation (B-9) is reduced to

$$B_1 = \frac{1 - r_0}{2 r_0}. \quad \text{(B-11)}$$

APPENDIX C

ANALYTICAL EXPRESSIONS FOR B_2 AND C_2

In this Appendix, we derive the analytical expressions of B_2 and C_2 given in the phase-domain generalized moveout approximation (9) for P–SV-waves in a homogeneous vertical transverse isotropy (VTI) layer. The general expressions for B_2 and C_2 are given by equations (11) and (12), i.e.,

$$B_2 = \frac{P \upsilon_n^2 \tau_0^2 + \hat{\tau} \hat{\tau}'}{P \upsilon_n^2 \left(\tau_0^2 - \hat{\tau}^2 + P \hat{\tau} \hat{\tau}'\right)} + \frac{A \tau_0^2 P^2 \upsilon_n^2}{\tau_0^2 (1 - P^2 \upsilon_n^2) - \hat{\tau}^2}, \quad \text{(C-1)}$$

$$C_2 = \frac{(P \upsilon_n^2 \tau_0^2 + \hat{\tau} \hat{\tau}')^2}{P^2 \upsilon_n^4 (\tau_0^2 - \hat{\tau}^2 + P \hat{\tau} \hat{\tau}')^2} + \frac{2 A \tau_0^2}{\tau_0^2 (1 - P^2 \upsilon_n^2) - \hat{\tau}^2}, \quad \text{(C-2)}$$

where $\hat{\tau} = \tau_c(P)$ and $\hat{\tau}' = \frac{d\tau_c}{dp}(P)$ denote the intercept of travel time and its derivative with respect to the horizontal slowness P for the non-zero offset reference ray, respectively.

From equation (8) and equations (A-6)–(A-11), we derive the traveltime intercept τ as follows:

$$\begin{aligned}\tau(p) &= (t_P - px_P) + (t_S - px_S) \\ &= t_{P0}\sqrt{1 - p^2\alpha_0^2(1 + a_0 + S(p))} \\ &\quad + \frac{t_{P0}}{r_0}\sqrt{1 - r_0^2 p^2\alpha_0^2(1 + b_0 - S(p))}.\end{aligned} \tag{C-3}$$

Taking the first-order derivative of equation (C-3) with respect to horizontal slowness p leads to the expression for τ', i.e.,

$$\begin{aligned}\tau'(p) &= -(x_p + x_s) \\ &= -p\alpha_0^2 t_{p0}\frac{1 + a_0 + S(p) + H(p)}{\sqrt{1 - p^2\alpha_0^2(1 + a_0 + S(p))}} \\ &\quad - p\alpha_0^2 r_0 t_{p0}\frac{1 + b_0 - S(p) - H(p)}{\sqrt{1 - p^2\alpha_0^2 r_0^2(1 + b_0 - S(p))}}.\end{aligned} \tag{C-4}$$

In equations (C-3) and (C-4), functions $H(p)$ and $S(p)$ are given by equations (A-12)–(A-14) of Appendix A.

We consider the limit case that the non-zero horizontal slowness reference ray becomes the horizontal ray. In this case, the horizontal slowness p becomes the maximum horizontal slowness for P-waves in a VTI medium given by equation (B-5). Therefore, equations (C-3) and (C-4) become

$$\hat{\tau} = \lim_{p\to p_h}\tau(p) = \frac{t_{p0}}{r_0}\sqrt{\frac{1 + a_0 + r_0^2(-1 + b_0 r_0^2)}{1 + a_0 + b_0 r_0^2}}, \tag{C-5}$$

$$\hat{\tau}' = \lim_{p\to p_h}\tau'(p) = \infty. \tag{C-6}$$

By substituting equations (C-5) and (C-6) into equations (C-1) and (C-2), we obtain the analytical expressions for B_2 and C_2 given by

$$B_2 = \frac{(1 + r_0)(1 + a_0 + (1 + b_0)r_0^2)}{2r_0(1 + a_0 + (1 + b_0)r_0)}, \tag{C-7}$$

$$C_2 = \frac{(1 + r_0)^2(1 + a_0 + b_0 r_0^2)}{(1 + a_0 + (1 + b_0)r_0)^2}. \tag{C-8}$$

For the special case of an isotropic medium ($a_0 = b_0 = 0$), equations (C-7) and (C-8) are reduced to

$$B_2 = \frac{1 + r_0^2}{2r_0}, \tag{C-9}$$

$$C_2 = 1. \tag{C-10}$$

APPENDIX D

MOVEOUT COEFFICIENTS FOR P–SV-WAVES IN A FACTORIZED VTI MODEL

For a factorized vertical transverse isotropy (VTI) model, it is often assumed that P- and SV-wave vertical velocities are linearly increased with the depth z, and the ratio r_0 between P- and SV-wave vertical velocities and Thomsen (1986) parameters ε and δ are constant within this layer. Under these assumptions, the vertical velocities of P- and SV-waves are represented by equations (32) and (33), i.e.,

$$\alpha(z) = \alpha_0\left(1 + \frac{\gamma - 1}{H}z\right), \tag{D-1}$$

$$\beta(z) = r_0\alpha_0\left(1 + \frac{\gamma - 1}{H}z\right), \tag{D-2}$$

where α_0 denotes the P-wave vertical velocity at the surface, H is the thickness of the factorized VTI model, and $\gamma = \alpha(H)/\alpha_0$ is the ratio between the vertical P-wave velocity to the bottom and the vertical P-wave to the top of the layer.

The P–SV-wave travel time t is represented by an integral with respect to the horizontal slowness p, i.e.,

$$t(p) = xp + \int_0^H (q_P(z) + q_S(z))dz, \tag{D-3}$$

where subscripts "P" and "S" denote P- and SV-waves, respectively.

In equation (D-3), the range of the horizontal slowness p is given by

$$0 \le p \le \frac{1}{\gamma\alpha_0\sqrt{1 + a_0 + b_0 r_0^2}}, \tag{D-4}$$

where $a_0 = 2\delta$ and $b_0 = 2(\varepsilon - \delta)/r_0^2$ are normal moveout factors for the P- and SV-waves, respectively.

From equation (D-3), we derive the offset $x(p)$ of P–SV-waves, i.e.,

$$x(p) = -\int_0^H (\partial_p q_P(p, z) + \partial_p q_S(p, z))dz, \tag{D-5}$$

where ∂_p denotes the first-order partial derivative with respect to the horizontal slowness p.

We expand the P-wave vertical slowness q_P at the zero-offset ray into a series in terms of the horizontal slowness p, i.e.,

$$q_P(p,z) = c_{P0}(z) + c_{P2}(z)p^2 + c_{P4}(z)p^4 + \cdots, \quad \text{(D-6)}$$

where

$$c_{P0}(z) = \frac{1}{\alpha(z)}, \quad \text{(D-7)}$$

$$c_{P2}(z) = -\frac{1}{2}\alpha(z)(1+a_0), \quad \text{(D-8)}$$

$$c_{P4} = -\frac{1}{8}(1+a_0)^2\alpha_0^3 - \frac{1}{2}\frac{b_0 r_0^2\alpha_0^3(1+a_0-r_0^2)}{1-r_0^2}. \quad \text{(D-9)}$$

Similarly, the Taylor expansion of the SV-wave vertical slowness q_S is obtained as follows:

$$q_S(p,z) = c_{S0}(z) + c_{S2}(z)p^2 + c_{S4}(z)p^4 + \cdots, \quad \text{(D-10)}$$

where

$$c_{S0}(z) = \frac{1}{r_0\alpha(z)}, \quad \text{(D-11)}$$

$$c_{S2}(z) = -\frac{1}{2}r_0\alpha(z)(1+b_0), \quad \text{(D-12)}$$

$$c_{S4} = -\frac{1}{8}r_0^3(1+b_0^2)\alpha_0^3 + \frac{b_0 r_0^3(1+2a_0-r_0^2)\alpha_0^3}{4(1-r_0^2)}. \quad \text{(D-13)}$$

By substituting equations (D-6) and (D-10) into equation (D-5) and calculating the inverse series of equation (D-5), we finally derive the following:

$$p(x) = -\frac{x}{2d_2} + \frac{d_4x^3}{4d_2^4} + \frac{3(-4d_4^2 + d_2d_6)x^5}{32d_2^7} + \cdots, \quad \text{(D-14)}$$

with

$$d_0 = \frac{H(1+r_0)\ln\gamma}{r_0\alpha_0(\gamma-1)}, \quad \text{(D-15)}$$

$$d_2 = -\frac{1}{4}H(1+a_0+r_0+b_0r_0)\alpha_0(1+\gamma), \quad \text{(D-16)}$$

$$\begin{aligned} d_4 = &-H(a_0^2(1+r_0) + 2a_0(1+r_0+2b_0r_0^2) \\ &+ (1+r_0)(1-2b_0(-2+r_0)r_0^2 + r_0^3 + b_0^2r_0^3)) \\ &\times \alpha_0^3(1+\gamma+\gamma^2+\gamma^3)/(32(1+r_0)), \end{aligned} \quad \text{(D-17)}$$

$$\begin{aligned} d_6 = &-H\alpha_0^5(1+\gamma+\gamma^2+\gamma^3+\gamma^4+\gamma^5) \\ &\times(a_0^3(1+r_0)^2 + a_0^2(3+6r_0+(3+4b_0)r_0^2+8b_0r_0^3) \\ &+ a_0(3+6r_0+(3+8b_0)r_0^2+16b_0r_0^3 \\ &+ 4b_0(1+2b_0)r_0^4 + 4(-1+b_0)b_0r_0^5) \\ &+ (1+r_0)(1+r_0+4b_0r_0^2+4b_0r_0^3+8b_0^2r_0^4 \\ &+ (-1+b_0)^2(1+b_0)r_0^5 + (-1+b_0)^2(1+b_0)r_0^6)) \\ &/(96(1+r_0)^2). \end{aligned} \quad \text{(D-18)}$$

From equations (D-3), (D-6), (D-10), and (D-14), it follows that the P–SV-wave moveout expansion is written in the following form:

$$t^2(x) = t_0^2 + \frac{x^2}{\upsilon_n^2} + \frac{Ax^4}{2t_0^2\upsilon_n^4} + \frac{Ex^6}{6t_0^4\upsilon_n^6} + \cdots, \quad \text{(D-19)}$$

where

$$t_0 = \frac{H(1+r_0)\ln\gamma}{r_0\alpha_0(\gamma-1)}, \quad \text{(D-20)}$$

$$\upsilon_n^2 = \alpha_0^2\frac{r_0(1+a_0+r_0+b_0r_0)(\gamma^2-1)}{2(1+r_0)\ln\gamma}, \quad \text{(D-21)}$$

$$A = \frac{1}{2} - \frac{((1+a_0)^2(1+r_0) + 4(1+a_0)b_0r_0^2 + (1+b_0)^2r_0^3 + (-1+b_0)^2r_0^4)(1+\gamma^2)\ln\gamma}{2r_0(1+a_0+r_0+b_0r_0)^2(\gamma^2-1)}, \quad \text{(D-22)}$$

$$E = \frac{3d_0(d_2^2d_4 + 4d_0d_4^2 - d_0d_2d_6)}{2d_2^4}. \quad \text{(D-23)}$$

Note that d_j, $j = 0, 2, 4, 6$, in equation (D-23) are given by equations (D-15)–(D-18).

For the special case of a homogeneous VTI layer ($\gamma = 1$), we obtain the corresponding coefficients given by the following:

$$t_0 = \frac{H(1+r_0)}{r_0\alpha_0}, \quad \text{(D-24)}$$

$$\upsilon_n^2 = \alpha_0^2\frac{r_0(1+a_0+(1+b_0)r_0)}{1+r_0}, \quad \text{(D-25)}$$

$$A = -\frac{(1+a_0+(-1+b_0)r_0^2)^2}{2r_0(1+a_0+(1+b_0)r_0)^2}, \quad \text{(D-26)}$$

$$E = 3(1+a_0+(b_0-1)r_0^2)^2((1+a_0)^2+(1+a_0)(a_0-b_0)r_0 + 2(1+a_0)(3b_0-1)r_0^2+(1+b_0)(b_0-a_0)r_0^3 + (1+(b_0-6)b_0)r_0^4)/(4r_0^2(1+a_0+(1+b_0)r_0)^4). \quad \text{(D-27)}$$

APPENDIX E

MOVEOUT COEFFICIENTS FOR P–SV-WAVES IN A MULTI-LAYERED VTI MODEL

For P–SV-waves in an N-layer vertical transverse isotropy (VTI) model, the travel time is represented in terms of the horizontal slowness, i.e.,

$$t(p) = xp + \sum_{i=1}^{N} (z_i q_{Pi}(p) + z_i q_{Si}(p)), \quad \text{(E-1)}$$

where z_i denotes the thickness of the ith VTI layer, and q_{Pi} and q_{Si} denote the vertical slowness of P- and SV-waves in the ith VTI layer. The expressions for q_{Pi} and q_{Si} are similar to the ones given by equation (A-1).

From equation (E-1), we derive the source–receiver offset of P–SV-waves, i.e.,

$$x(p) = -\sum_{i=1}^{N} \left(z_i \partial_p q_{Pi}(p) + z_i \partial_p q_{Si}(p)\right), \quad \text{(E-2)}$$

where ∂_p denotes the first-order partial derivative with respect to the horizontal slowness p.

We expand the sum of terms given in equation (E-1) into a series with respect to the horizontal slowness p at $p = 0$, i.e.,

$$\sum_{i=1}^{N} (z_i q_{Pi}(p) + z_i q_{Pi}(p)) = d_0 + d_2 p^2 + d_4 p^4 + d_6 p^6 + \cdots, \quad \text{(E-3)}$$

where

$$d_j = \sum_{i=1}^{N} c_j^i, \quad j = 0, 2, 4, 6, \ldots \quad \text{(E-4)}$$

with

$$c_0^i = t_{P0i}\left(1 + \frac{1}{r_{0i}}\right), \quad \text{(E-5)}$$

$$c_2^i = -\frac{1}{2} t_{P0i} \alpha_{0i}^2 (1 + a_{0i} + (1 + b_{0i}) r_{0i}), \quad \text{(E-6)}$$

$$c_4^i = -t_{P0i}\alpha_{0i}^4((1+a_{0i})^2(1+r_{0i}) + 4(1+a_{0i})b_{0i}r_{0i}^2 + (1+b_{0i})^2 r_{0i}^3 + (-1+b_{0i})^2 r_{0i}^4)/(8(1+r_{0i})), \quad \text{(E-7)}$$

$$c_6^i = -t_{P0i}\alpha_{0i}^6((1+a_{0i})^3 + 2(1+a_{0i})^3 r_{0i} + (1+a_{0i})^2(1+a_{0i}+4b_{0i})r_{0i}^2 + 8b_{0i}(1+a_{0i})^2 r_{0i}^3 + 4b_{0i}(1+a_{0i})(1+2b_{0i})r_{0i}^4 + (1+b_{0i}(-1+4a_{0i}(-1+b_{0i})+b_{0i}(7+b_{0i})))r_{0i}^5 + 2(-1+b_{0i})^2(1+b_{0i})r_{0i}^6 + (-1+b_{0i})^2(1+b_{0i})r_{0i}^7)/(16(1+r_{0i})^2). \quad \text{(E-8)}$$

Here the subscript "i" denotes the ith VTI layer, t_{P0i} denotes the one-way vertical travel time of P-waves in the ith VTI layer, and α_{0i} denotes the P-wave vertical velocity for the ith VTI layer. r_{0i} denotes the ratio between vertical velocities of P- and SV-waves for the ith VTI layer, and $a_{0i} = 2\delta_i$ and $b_{0i} = 2(\varepsilon_i - \delta_i)/r_{0i}^2$ are NMO factors of P- and SV-waves in the ith VTI layer, where ε_i and δ_i are Thomsen (1986) parameters defined for the ith VTI layer.

From equations (E-1)–(E-3), it follows that the P–SV-wave moveout expansion is written in the following form:

$$t^2(x) = t_0^2 + \frac{x^2}{\upsilon_n^2} + \frac{Ax^4}{2t_0^2\upsilon_n^4} + \frac{Ex^6}{6t_0^4\upsilon_n^6} + \cdots, \quad \text{(E-9)}$$

where the zero-offset two-way travel time t_0, normal moveout velocity υ_n, coefficients A and E are given by

$$t_0 = d_0, \quad \text{(E-10)}$$

$$\upsilon_n^2 = -\frac{2d_2}{d_0}, \quad \text{(E-11)}$$

$$A = \frac{1}{2} + \frac{d_0 d_4}{d_2^2}, \quad \text{(E-12)}$$

$$E = \frac{3d_0(d_2^2 d_4 + 4d_0 d_4^2 - d_0 d_2 d_6)}{2d_2^4}. \quad \text{(E-13)}$$

Here, $d_i, i = 0, 2, 4, 6$, are given by equations (E-4)–(E-8).